My talk is borrowed in spirit from David Hilbert who, at age 38 in 1900, defined 23 mathematical challenges that more than a century later, while all not fully resolved, shaped the course of advances in that field from then until now.

He gave grand definitions. Instead, I point to challenges that address the present crisis in satisfactory understanding and prediction of acoustics in shallow water environments.
My paper will mainly use test results in shelf waters that are about 100 m deep.
THE CANONICAL MODEL OF THE ACOUSTICS DUCT FOR SHELF WATERS

The two-dimensional plane shown idealizes the cylindrical volume containing the sound semi-infinite atmosphere

water

semi-infinite basement

For slowly-varying depth- and range-dependent values of the material properties of the layers, acoustic transmission in a shelf duct can be readily calculated. **Despite the stark simplicity of the canonical model, it is surprisingly informative.** Later caveats, however, underlie the challenges to fuller understanding of shallow water (SW) acoustics on the shelf.
THE REAL ACOUSTICS DUCT FOR SHELF WATERS HAS DEFECTS

\[ \frac{\Delta c_i}{c} \sim 2 \times 10^{-3}, \ d_i \sim 10 \times 10^3 \text{m} \quad :::: \quad \frac{\Delta c_f}{c} \sim 2 \times 10^{-1}, \ d_f \sim 10^{-2} \times 10^{-2} \text{m} \]

\[ \sigma, \ \Lambda \quad (\text{m}) \quad 1, \ 10 \quad 10^{-2}, \ 1 \quad 1, \ 10^3 \quad 10, \ 10^4 \]

Height & length scales \((\sigma, \Lambda)\) of the duct’s interfaces, and sound speed-contrasts & length scales \((\Delta c, d)\) of the volume inhomogeneities lead to **scattering** and to **spatial decoherence** of the signal and noise. Duct defects can affect the mean values of transmission and noise via **net out-of-plane scattering losses**. Also, the defects, and motions of the source, of the receiver, of the air/water interface, and of the water inclusions contribute to **temporal decoherence**.
ECS ASIAEX 3 June 2001
Systematic Transmission Data vs Bearing: A Breakthrough in Test Design

from Abbot, Dyer, & Emerson, 2006
The polar plots are vs source bearing $\beta$, for normalized receiver depths in the usual reading order: $d/D = 0.33$ to 0.82 in 14 equal increments.

Caution: some data are missing.
With the anomaly at $\beta \approx 030^0$ excluded, the $\sigma_{TL}$ is approximately independent of $\beta$ (i.e., isotropic), and of value: $\sigma_{TL} \approx 2$ dB (green circles)

$\sigma_{TL}$, dB (7 to 0 dB)
ECS ASIAEX, 400 Hz
$R = 30$ km
Red: Data Disparate in Time
Green: $\langle \sigma_{TL} \rangle$ over $\beta$, except $\beta \approx 030^0$

from Abbot, Dyer, & Emerson.2006
ASIAEX (June 2001)
3 Radial Tracks and Circle Track

HEP (Sept. 1997), 4 Radial Tracks

With exclusion of the anomaly, and from data in range-bins, we obtained $<\mu_{TL}>_\beta$, the bearing-averaged transmission mean vs range.

The canonical model can robustly predict $<\mu_{TL}>_\beta$, defects not withstanding!

from Abbot, Dyer, & Emerson, 2006
Mean ($<\mu_{TL}>_{\beta}$ on left) and Standard Deviation ($<\sigma_{TL}>_{\beta}$ on right)
Measured vs Normalized Depth, ECS, ASIAEX 2001

400 Hz Octave, $000^0 \leq \beta \leq 360^0$ excluding the Anomaly at $\beta \approx 030^0$
Assimilated to $R = 30$ km and $d_s = 50$ m

from Abbot, Dyer, & Emerson 2006
Assertion: While transmission data show $\mu_{TL}$ to be dependent on source/receiver bearing $\beta$ on the shelf (see Slide 6), the canonical duct can be used to predict, as supported by Slide 8, sound transmission for shelf waters averaged over bearing ($<\mu_{TL}>_\beta$). Some, but not all, of the crisis in understanding is thus overcome.

Challenge 1: What are the physical limits of this assertion in relation to the observable properties of the ocean? As examples:

- Does this assertion hold for all combinations of duct defects?
- Do transmission data for all broad shelf waters behave similarly?
- Do shelf currents, or eddies shed from major continent-hugging circulations such as the Gulf Stream, play a role?
- Because they can cause out-of-plane losses, are rough sea surfaces or fish schools, truly negligible in determining the mean $<\mu_{TL}>_\beta$?
CONSIDERING OCEAN ENVIRONMENTAL DEFECTS

Assertion: Understanding and predicting the mean ($<\mu_{TL}>_\beta$) is conceptually easy via the canonical model. But the absence of defects in this model has the effect that it cannot correctly predict other highly-desired properties of the sound field, including fluctuations around the mean (e.g., $\sigma_{TL}$), scattering (e.g., reverberation), scattering-induced losses, and spatial and temporal decoherence.

We consider next some defects, as revealed by SW experiments. These couple acoustics to data from two ocean environmental disciplines (physical oceanography and ocean geology/geophysics), but this is enough to illustrate the quest for better and fuller understanding.
The Internal Tide: An In-Water Macroscale Defect
Well Resolved by Thermistor Data
ECS ASIAEX, 3 June 2001, at Station M, 20 sensor Chain, Shi Yan-3, (Peng et al)

Weak $g_H$: refracts mostly in-plane; no anomaly

In overall this is a "macroscale" defect; the major features are $> 100$ km in length.

Strong horizontal gradient ($g_H$): sound refracts in & out-of-plane, and explains the anomaly at $\beta \approx 030^0$
Standard Deviation ($\sigma_{TL}$) vs Normalized Depth
Excluding (left) and Only for the Anomaly at $\beta \approx 030^\circ$ (right)

ECS ASIAEX 2001, 400 Hz Octave, $R = 30$ km

The macroscale anomaly is spatially and temporally rare (Finette, 2003) but, when observed, has a large standard deviation sharply dependent on depth

from Abbot, Dyer, & Emerson, 2006
Microscale Contrast Volumes, $n^2$ vs R and d, are Numerous but Weak, and Poorly Resolved by Thermistor Data

Vertically Quantized, Rigid Advection Landward (assumed at 0.9 m/s)
Oppositely Directed Acoustic Propagation Wavevector (at $\approx 1500$ m/s)

Based on Peng et al

$n^2 = (\Delta c/c)^2$
Histogram of TL, Radial Track (F/M), $\beta = 300^0$

ECS ASIAEX, $d_R = d_S = 50\ m$

Histogram of Differences, ASIAEX Down Slope (F to M), $d/D = 0.48$

$N = 30$
$\mu = -1.0\ dB$
$\sigma = 2.2\ dB$
Duration: 90 min.
$R \leq 30\ km$
Delineation of 2D **macroscale** features from temperature data vs depth and time, as in standard physical oceanography, provides only part of the fundamental information needed to understand real duct acoustics. Also, there are limitations to this standard:

- A **macroscale** temperature slice as shown in Slide 12, being 2D, can answer questions only partially on the 3D acoustic field.
- But the temperature field can be measured in 3D with use of mobile sensors and assimilative modeling; this recently has been shown to be feasible (Gawarkiewicz, 2007), and hopefully will become standard.
- 3D **microscale** features from temperature data, however, are not measured at all or, if so, are too poorly resolved for acoustic analyses.
- Other water-dynamical defects, such as the rough sea surface or dense fish schools, each a defect class with strong acoustic contrast (Makris et al, 2006), are typically not measured simultaneously with acoustic tests.
Thickness of the two ASIAEX Sedimentary Layers

**First Layer.** Between water and second layer, $\mu = 1.6 \text{ m}$

**Second Layer.** Between first layer and basement, $\mu = 10.8 \text{ m}$

from Bartek data
THE BASEMENT INTERFACE IN THE ECS

Sediment-covered ancient river-bed

Looking ~ NW

Bartek ASIAEX Basement Depth from Sea Surface, m

East Longitude 127.5 29
North Latitude 29 29.2 29.4 29.6 29.8

Data assembled by Emerson & Dyer
Data assembled by Emerson

Unresolved microscale features $O(1\text{m})$ can affect $\sigma$

Values from 3$^\circ$ to 10$^\circ$ can be important, but are not shown here

Isolated macroscale features $O(5\text{ km})$ can affect $\mu$
One-Dimensional Wavenumber Spectrum $S_h(k_Y)$ of Basement Depth

(Depth Track 15, ECS Data)

Power-law exponent $\gamma$
Slope = -2.4
$r^2 = 0.66$

$S_h(k_Y) = A^2 (k_Y)^{-\gamma}$
$\mu_\gamma = -2.6$, $\sigma_\gamma = 0.4$
for all 11 $k_Y$
basement tracks

$\mu_\gamma = -2.1$ for 4 tracks
between layer 1 & 2

The arrow extends
toward the acoustic
domain, beginning
at $f = 10$ Hz, for
which roughness
data are, at best,
extrapolations

Analysis by Emerson of Bartek’s data
2D Wavenumber Spectra of the two ASIAEX Bottom Interface Depths* 
Appear Approximately Horizontally Isotropic, with $O(10)$ Larger Spectral 
Density for the Basement Interface

* Inferred from Sediment Layer Thickness

Between layer 1 and 2

Between layer 2 and basement

Abbot, Dyer, & Emerson, 2006

* Inferred from Sediment Layer Thickness

from Bartek data
Acoustics and Ocean Geology/Geophysics

• Pseudo-3D shelf-bottom features, from standard ocean geology/geophysics, also pertain to real duct acoustics. Although the sediment-covered ancient river-bed is a lineal feature, the measured 2D macroscale roughness appears to be more isotropic than anisotropic.

• Two rough interfaces were delineated in the ECS shelf bottom, the first of $O(1 \text{ m})$ into the bottom, and the second at the basement of $O(10 \text{ m})$ deeper. The first is probably not rough enough to significantly disrupt evanescent waves in the bottom (and thus not affect low-angle bottom acoustic reflectivity), and the second carries macroscale scars from its tectonic and erosive history that could scatter sound strongly, but mainly for angles larger than critical (Abbot, Dyer, & Emerson, 2006).

• Roughness spectra, and the derived roughness slopes, show that macroscale flat sections are numerous, more so for the interface between layer 1 and 2, than for the basement. Also, while basement microscale features at the high wavenumbers pertaining to the acoustic domain can be extrapolated from low wavenumber data, this comes with uncomfortable risk.
Could Macroscale Defect Length-Scales $\Lambda_1$ (mean distance between neighboring defects) and $\Lambda_2$ (mean size of defects) be Estimated from these Data?

**Measured Transmission**
(Data Shown Earlier)

The two arrivals, 10 s apart, are poorly correlated, more so in NW sectors. At $R = 30$ km, one could dismiss the sediment-covered ancient river-bed as an important scatterer (but not for $R < 10$ km).

Caution: some data are missing

Anomaly

All circles 18 dB radii

**TL, dB re 1m (82 to 64 dB)**

**ECS ASIAEX**

50 & 25m Source Depths

Circle Track, 400 Hz Octave

26.6 < $R$ < 33.9 km
Fluctuation Statistics and Inferred Defect Scales

• The 10 s interval between the First and Second transmission samples at each $\beta$ in the ECS circle is too large to define a temporal scale. Rather, in this interval, the source translated $\approx 51$ m or about 14 acoustic wavelengths. This is more than enough to invoke phase-random (pr) statistics for the $\beta$ sample-set that includes the 1$^{\text{st}}$ and 2$^{\text{nd}}$ arrivals, resulting in $\sigma_{\text{pr}} \approx 0.2$ dB; this decreases the observed $\sigma_{\text{TL}}$, a difference in this case of O(0.01) dB that is small enough to be ignored.

• The distance between the inferred defect peaks might suggest $\Lambda_1 \approx \text{O}(15 \text{ km})$, and the width of the peaks $\Lambda_2 \approx \text{O}(4 \text{ km})^*$. Either or both of these might be plausibly connected to macroscale defects caused by the internal tide, or by the bottom, or by other defect classes. The correlation between the 1$^{\text{st}}$ and 2$^{\text{nd}}$ arrivals is acceptable for an average of only 3/25 of the 14$^0$ bearing sectors; it is thus inappropriate to accept these scale inferences.

*The bearing interval $\Delta\beta \approx 14^0$ at $R = 30$ km has a circumferential period $\approx 7.3$ km, so that scales shorter than this cannot be resolved. That is, much finer sampling in $\beta$ is needed.
OASIS Mobile Acoustic Source (OMAS)

OMAS Characteristics
- Precision Clock
- Calibrated Sound Source
- Specialized Acoustic Transmissions
- LBL Tracking Systems

Field Applications
- SCORE Range 04
- South China Sea 05, ECS 06
- New Jersey Experiments 05, 06

Standard EMATT (LM Sippican)
- Length: 91.4 cm (36”)
- Diameter: 12.4 cm (4.9”)
- Weight: 10 kg (22 lbs)
- Battery Power: LiSO4

Operational Characteristics
- Depth: 23 – 183 m, ± 5m (75 - 600 ft)
- Speed: 1.5 – 4 m/s (3 - 8 Knots)
- Endurance: 3 to 6 hrs, Speed Dependent
- Launch: Handheld
Range-Whitened Transmission on the SCS Shelf, 2005, $D = 77 - 95$ m, $\Delta \Theta = 7$ hr
OMAS Sources 55 m, Moored Receiver 28.5 m, Replica Processed, $f = 900$ Hz

Data by Emerson, et al, 2007
For bearings excluding the front sectors, the mean $\langle \mu_{TL} \rangle_\beta$ is horizontally isotropic, and translationally invariant for at least 12 km along the isobaths.
Transmission Means in $5^0$ Sectors vs Bearing, at $R = 7.5$ km
NJ Shelf 2006, $D = 80 - 120$ m, $f = 900$ Hz, Replica Processed

The circumferential scale in each $\Lambda_2 \approx 1$ km.
Isotropy and translational invariance is preserved (outside the front sectors).

An underlying structure of the acoustic field is resolved with $5^0$ averaging. The environmental or other cause of this structure is as yet unknown.

Data by Abbot, Emerson, Gedney
Assertion: In shelf waters, real duct defects cause: 1) fluctuations around the mean transmission, with defect scale $\Lambda_2 \approx O(1 \text{ km})$, and 2) scattering in all directions.

Challenge 2: What is the physical explanation for the observed 1 km defect-scale? All SW defects referred to in this talk are plausible candidates, but none can be ruled out unequivocally with the environmental data acquired. On the other hand, is the observed $\Lambda_2$ scale of $O(1 \text{ km})$ simply the inherent period of the sound field, which itself is a complicated function of range?

Challenge 3: Defects are three-dimensional (3D) objects, and scattering itself is a 3D process. Thus, the 2D concepts now commonly in use for understanding and prediction need to be extended or replaced by 3D analytical/numerical approaches, such as surveyed by Robinson and Lee (1994), among others.
Challenge 4: Physical oceanography of shallow water, and ocean geology/geophysics of its bottom, have contributed, mainly by illuminating macroscale water-dynamics and bottom-roughness defects. But acoustics tests need to be supported also with quantitative real-time data on sea-surface roughness, below surface bubble clouds, and fish schools, etc., each of which relate to significant defects.

Assertion: Macroscale defects in each class can be treated as perturbations in, as examples, deterministic 3D adiabatic or 3D refractive Fresnel-tube formulations.

Challenge 5: Deterministic analysis of small defects (microscale down to wavelength-scale) is not affordable, either intellectually or fiscally, and thus would need to be abandoned in favor of stochastic analysis. (The next slide suggests that distributions of small defects could be treated as multiple source functions that represent random scatterers.)
“DEFECTS” IN AN AIR-CONDITIONING DUCT
An aside based on Dyer, 1958

• Transmission from a number, N, of δ-function spatially uncorrelated random acoustic sources was studied. These sources were distributed over the cross-section of an A/C duct of constant radius, r, and the field propagating in the duct was determined vs r.

• Conclusion: The lowest order modes (up to ~ ½ N) propagate with statistically equal acoustic intensity, i.e., are “energy-equipartitioned”, and are “statistically independent”. This theoretical conclusion agrees with measured fan-driven noise in an A/C duct data (Kerka, 1957), in which the equipartitioned modes combine to match the observed radial dependence. (The underlined properties are classical features of the dynamics of multi-degree systems.)
Assertion: In SW acoustics, microscale scattering defects act as $N$ spatially uncorrelated random sources, with each proportional to the local primary sound field. $N$ is the number of the more important small defects between receiver and source which, in most cases, would be local to the receiver. (The defects create a subsidiary 3D field of scattered acoustic waves, which are composed of $\frac{1}{2} N$ equipartitioned modes, each with variance $\sigma_d^2$. In the limit of large $N$, $\sigma_d^2$ would be approximately independent of depth, range, and bearing.)

**Challenge 6:** What are the physical and computational limits that define the boundaries between macroscale and microscale defects?

**Challenge 7:** How should tests be designed, and data be analyzed, to distinguish out-of-plane losses due to scattering, from the commonly considered losses in the water and in the sediments?
Challenge 8: Because high-resolution temporal processing is already common in ocean acoustics, new or improved analytical/numerical tools also are needed to provide predictions in the time-domain. Path analysis comes to mind as a 3D possibility. Also the forward and backscattering full-wave analyses of Frankenthal & Beran (2006) for sound in a square duct, with time-dependent volume defects, could set the path toward analysis of the real SW duct. (For energy transport, a square duct in essence is a range-whitened version of the cylindrical duct herein designated as the canonical model.)
Summary Perspectives on the Acoustics of Shelf Waters

- Mean transmission of a sound wave in a shelf-water duct is robustly predicted by the canonical model; it represents the mean ocean environment, that is, with all defects erased or smoothed out.
- Other than those due to phase-random summations of the sound field itself, transmission fluctuations are caused by defects; for most signal-processing methods, fluctuations due to the defects dominate.
- A large defect can be analyzed as a perturbation of the mean.
- A large number of small defects can be analyzed via the subsidiary sound field these defects generate by scattering.
- This subsidiary field also can limit temporal and spatial coherence for passive sonar, and is the field that directly limits active sonar.
- A reformulation of theoretical/numerical techniques that combine all of the foregoing would be ideally 4D (3D spatial, 1D temporal).
- Future at-sea tests would be ideally more inclusive of observations on all relevant defect classes, and of the diverse acoustic needs for the tests.
Assertion: The continental slope is acoustically more complicated than the continental shelf, even though the mean slope angle typically is $< 4^0$. Much less is known about defects on the slope compared to those on the shelf, and the retreat to averaging over bearing on the continental slope, to attain some degree of simplicity and general understanding, is patently a poor approach.
The Continental Slope is Significantly Different Acoustically Than the Shelf
Typical mean angles on the continental slope are 1° to 4°, with bottom defects similar to those on the shelf, but more dramatic and complicated by basement outcrops that dam the downward transport of sediments. Because a plane parallel to the isobaths has constant water depth, it is more like the real shelf duct, but transmission can be interrupted, or channeled, by outcrops.
Measured Transmission (peak replica processing) vs Receiver Track, OMAS Source on NJ Shelf (Black Track), Receiver on Slope or Shelf

\[ \text{Translational Invariance on Slope} \approx 4.5 \text{ km} \]

Data by Abbot, et al.

\[ N = 210 \quad \sigma = 2.1 \text{ dB} \]
\[ N = 81 \quad \sigma = 2.6 \text{ dB} \]
\[ N = 108 \quad \sigma = 2.2 \text{ dB} \]
Transmission on the Continental Slope in the Sea of Japan

February 1999 TL Test Geometry

May 1998 TL Test Geometry

Explosive Sources (BB)

High Resolution Source (NB)

SHAREM

HEP SUS 98

HEP SUS 99
HEP BB SOJ Slope Transmission, May 1998
SUS Sources, 400 Hz Octave, $d_s = d_r = 18$ m

**Range (km)**

- **TL, dB re 1m**
  - Cross (A-E)
  - Down (C-E)
  - Up (D-E)

- **R = 2.5 km:** 58 dB

- **Along the ~ 150 m Outcrop, A to E**
- **110 to 200 m over the Outcrop, C to E**
- **1000 to 350 m, up to the Outcrop, D to E**

From Abbot, Dyer, & Celuzza
SHAREM NB SOJ Slope Transmission

\( d_s = 8 \text{ m}, \quad d_r = 53 \text{ m}, \quad f = 3.5 \text{ kHz}, \quad \text{Replica Processed} \)

Red: Parallel, Blue: Normal, Down Slope, Green: Normal, Up Slope

\[ R = 2.5 \text{ kyd:} \]

\[ 73 \text{ dB} \]
Upon excluding the anomaly at $\beta \approx 030^0$, the mean $\mu_{TL}$ is approximately isotropic (green circles), but variations in $\beta$ remain.

$\mu_{TL}$, dB re 1m (66 to 80 dB)
ECS ASIAEX, R = 30 km
400 Hz Octave
Red: Data Disparate in Time
Green: $<\mu_{TL}>$ over $\beta$, except $\beta \approx 030^0$

from Abbot, Dyer, & Emerson, 2006