

# Extracting Green's functions from random noise and vibrations



*IRA DYER SYMPOSIUM*

*MIT*

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# OUTLINE

- INTRODUCTION
- UNDERWATER ACOUSTICS NOISE:
  - THEORY
  - EXPERIMENT
- SIGNAL PROCESSING (Sync and AEL or Arrays)
- SEISMOLOGY
- Other APPLICATIONS
  - STRUCTURAL ACOUSTICS
  - HUMAN BODY NOISE
- CONCLUDING REMARKS

# Green's Functions Estimate from "Noise"

## Background Theory and Data Realizations

### – Ultrasonics diffuse wavefields ( $\sim 1\text{MHz}$ )

- Lobkis and Weaver (*Phys. Rev. Lett.*, 2001), Derode et al. (JASA 2003), Larose et al. (JASA 2004), Malcolm et al. (*Phy. Rev. E.* 2004)

### – Structural Engineering ( $\sim 1\text{kHz}$ )

Farrar et al. (1997), Larose et al. (JASA 2006), Sabra et al. (JASA 2006)

### – Ocean ambient noise ( $\sim 100\text{Hz}$ )

- Roux and Kuperman (JASA, 2004), Roux et al. (JASA 2005), Sabra et al. (JASA, 2005)

### – Seismic ambient noise ( $< 1\text{Hz}$ )

- Aki (1957), Claerbout (Geophys. J. Int. 1968), Shapiro and Campillo (G.R.L., 2004, *Science* 2005), Snieder (*Phy. Rev. E.* 2004), Wapenaar et al. (Geophys. J. Int. 2004), Schuster et al. (Geophys. J. Int. 2004), Sabra et al. (G.R.L., 2005)

### – Helioseismology

- Duvall et al. (Nature, 1993), Claerbout & Rickett, (The Leading Edge 1999).

### -- Human Body Noise: Sabra et al, A. Phys. Ltrs (2007)

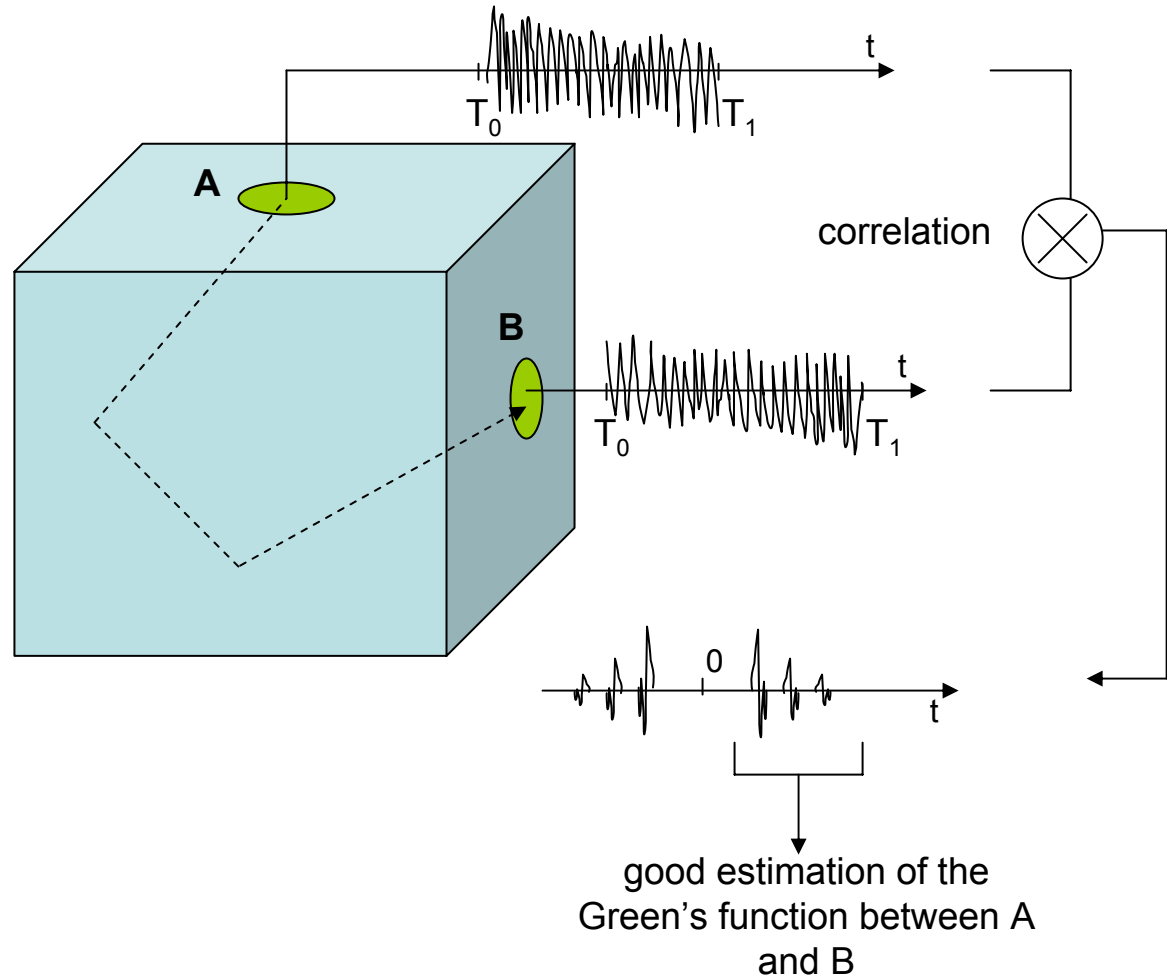
*Applications exist in a wide range of environments and frequency bandwidths because the physics driving this noise cross-correlation process remains similar.*

# Coherent signals from noise data

First experimental demonstration in ultrasonics (0.1 – 0.9 MHz)

R.L. Weaver & O.I. Lobkis, Phys. Rev. Lett., 2001

*“By cross-correlating ambient noise recorded at two locations, the Green’s function between these two locations can be reconstructed”. (J. Claerbout 1999, R. Weaver 2001.)*



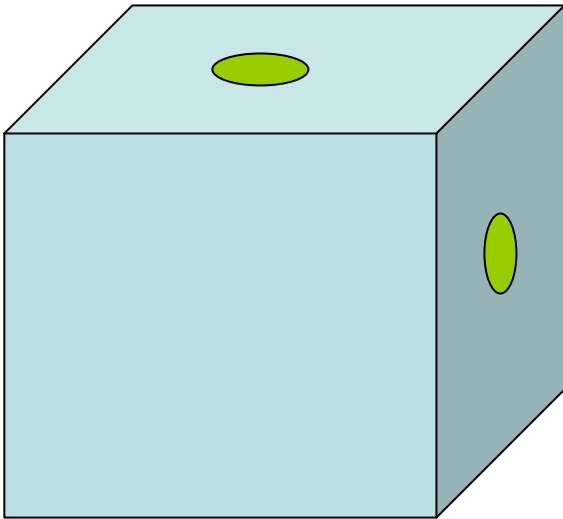
*HOW COME WEAVER  
CAN USE WHITE NOISE  
AND  
WE CAN'T?*

From array point of view: White noise is non-travelling, independent, unrelated noise hanging around each individual sensor.

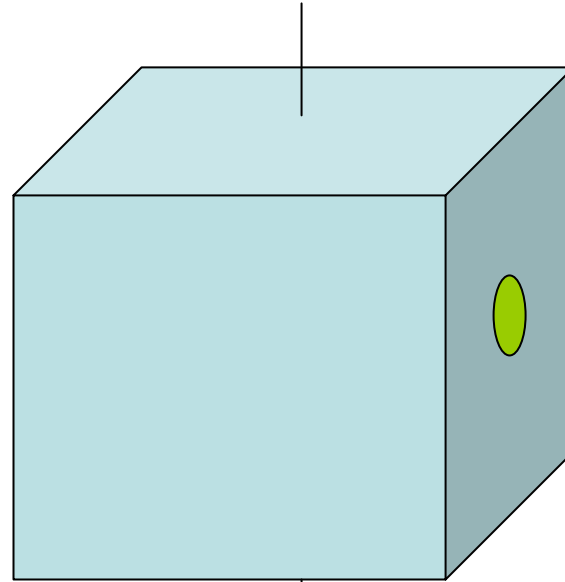
# ONE MAN'S WHITE NOISE...

CORRELATED

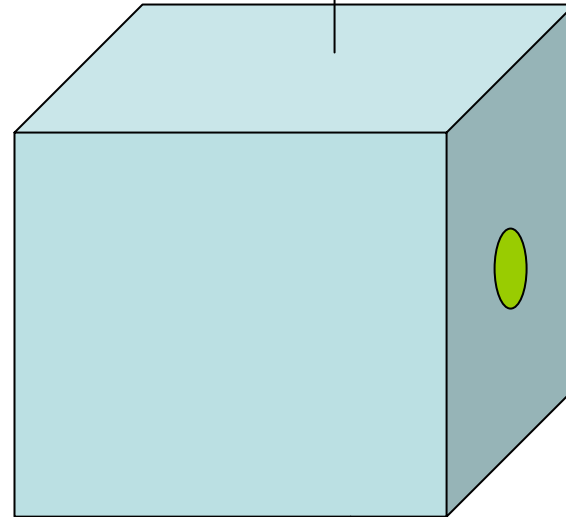
Weaver et al



ARRAY



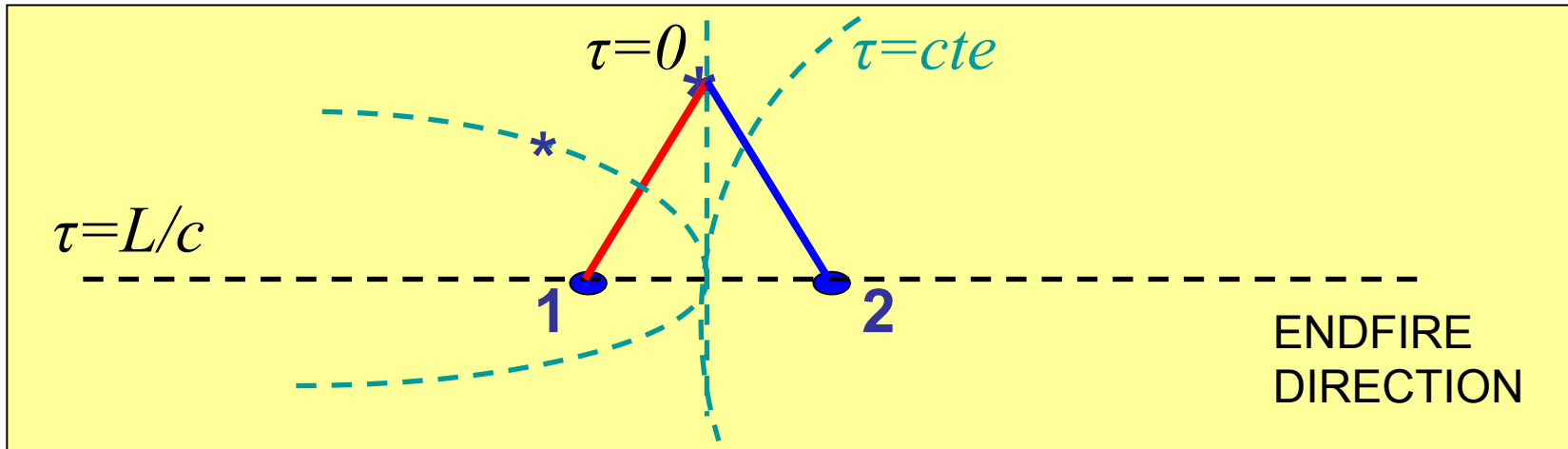
UNCORRELATED



*SENSOR  
NOISE*

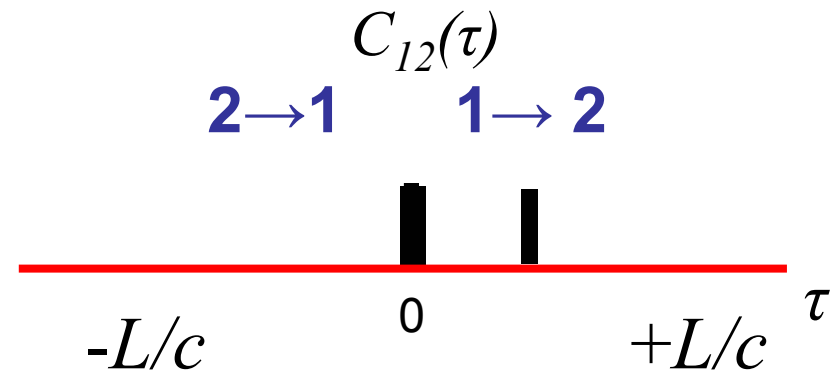
# Noise cross-correlation: Free space

$$C_{12}(\tau) = \int_{-\infty}^{\infty} P(\mathbf{r}_1, t) P(\mathbf{r}_2, t + \tau) dt.$$



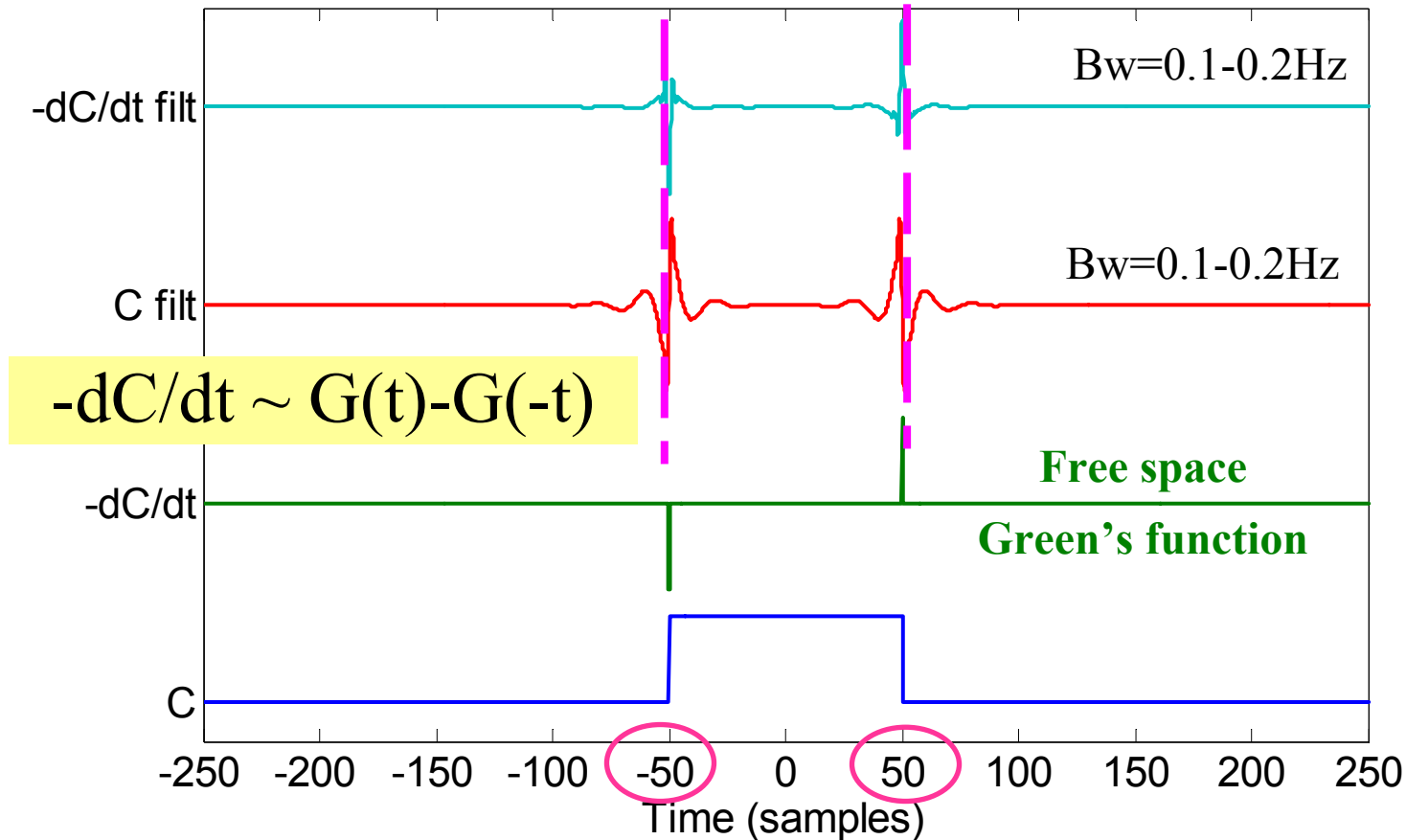
Noise sources yielding constant time-delay  $\tau$ , lay on same Hyperbola

$$\tau = \frac{|\mathbf{r}_2 - \mathbf{r}_s| - |\mathbf{r}_1 - \mathbf{r}_s|}{c} \leq \frac{L}{c}$$



Isotropic distribution of uncorrelated random noise sources

# C, dC/dt, band-limited signal

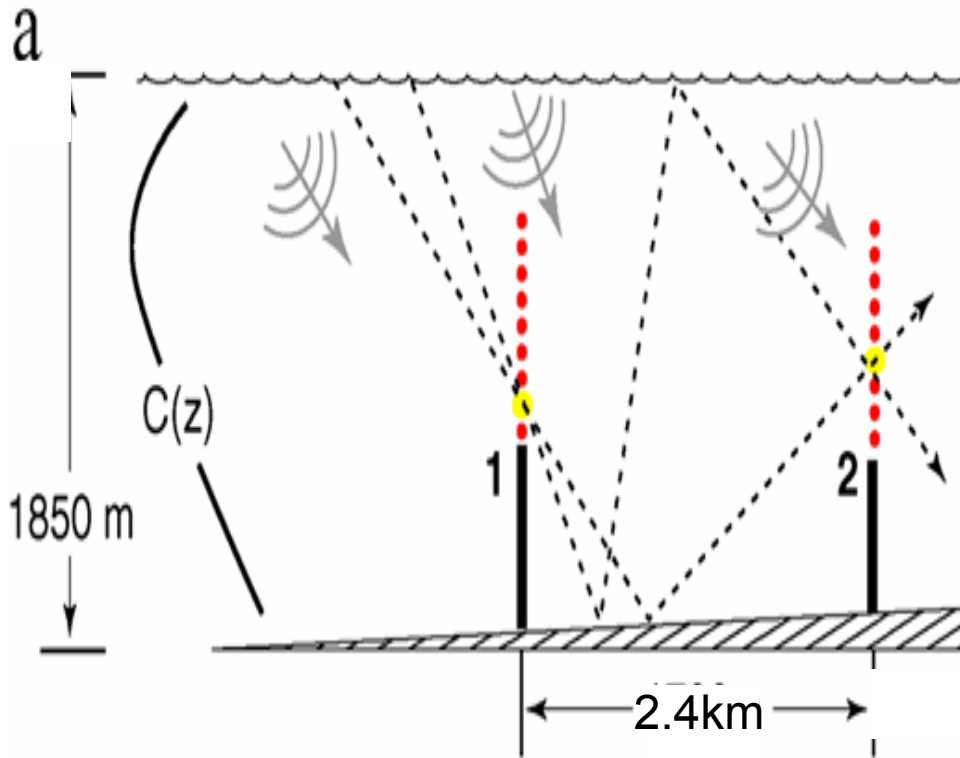


- With cross-correlation process the phase of the source signal is removed,  
→ Arrival time is given by the center of the pulse (envelope maximum)
- Isotropic noise distribution → Symmetric Correlation function.

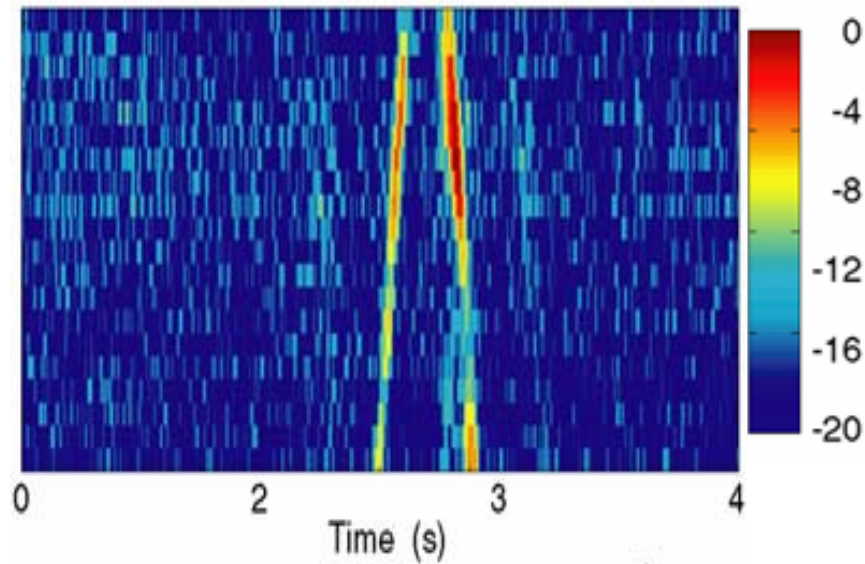


# Underwater Acoustics

(non-free space)



Experimental results (70 – 130 Hz)

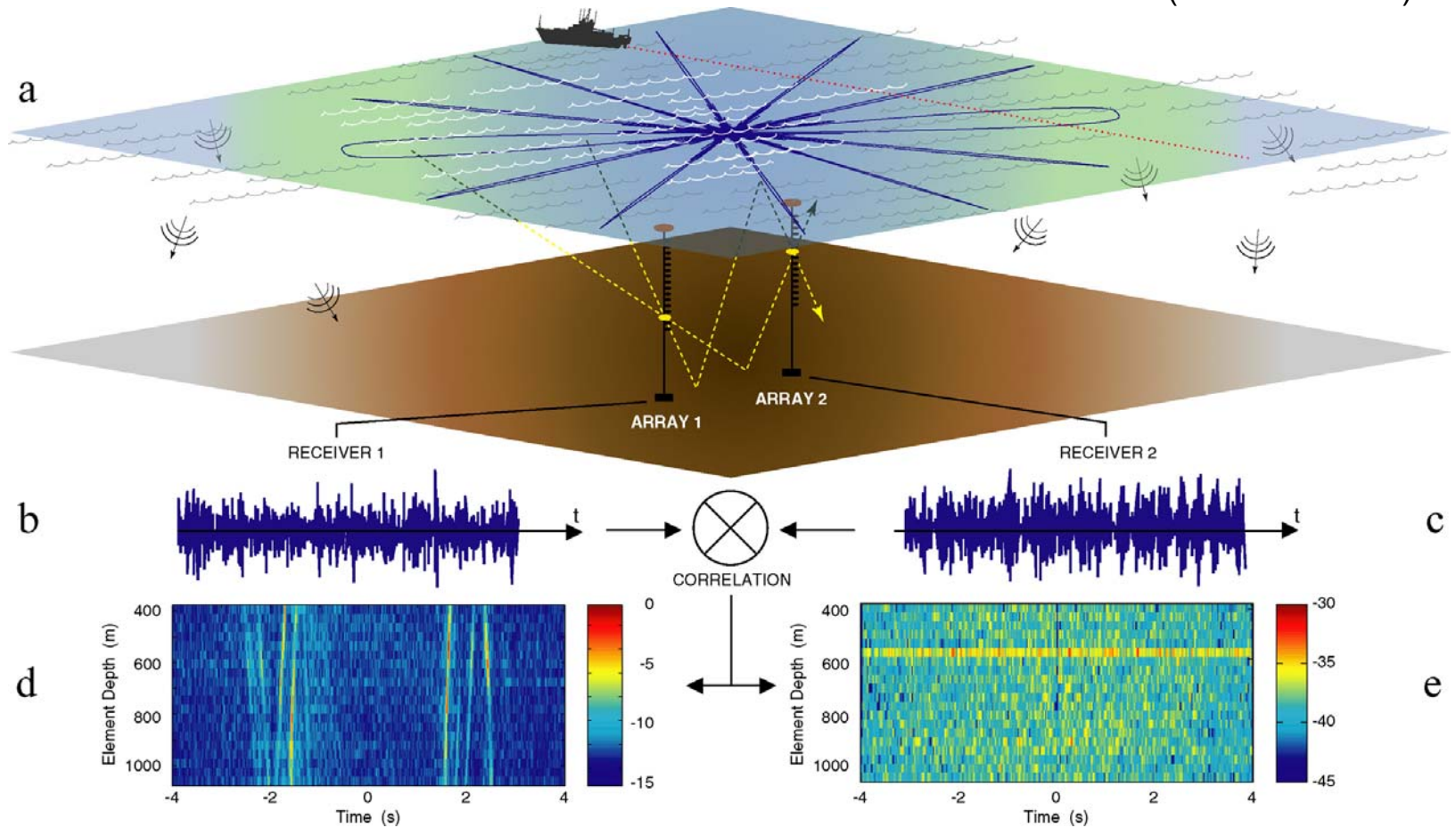


Noise events propagating through receivers 1 and 2 average-up coherently over the long-time in the cross-correlation function.

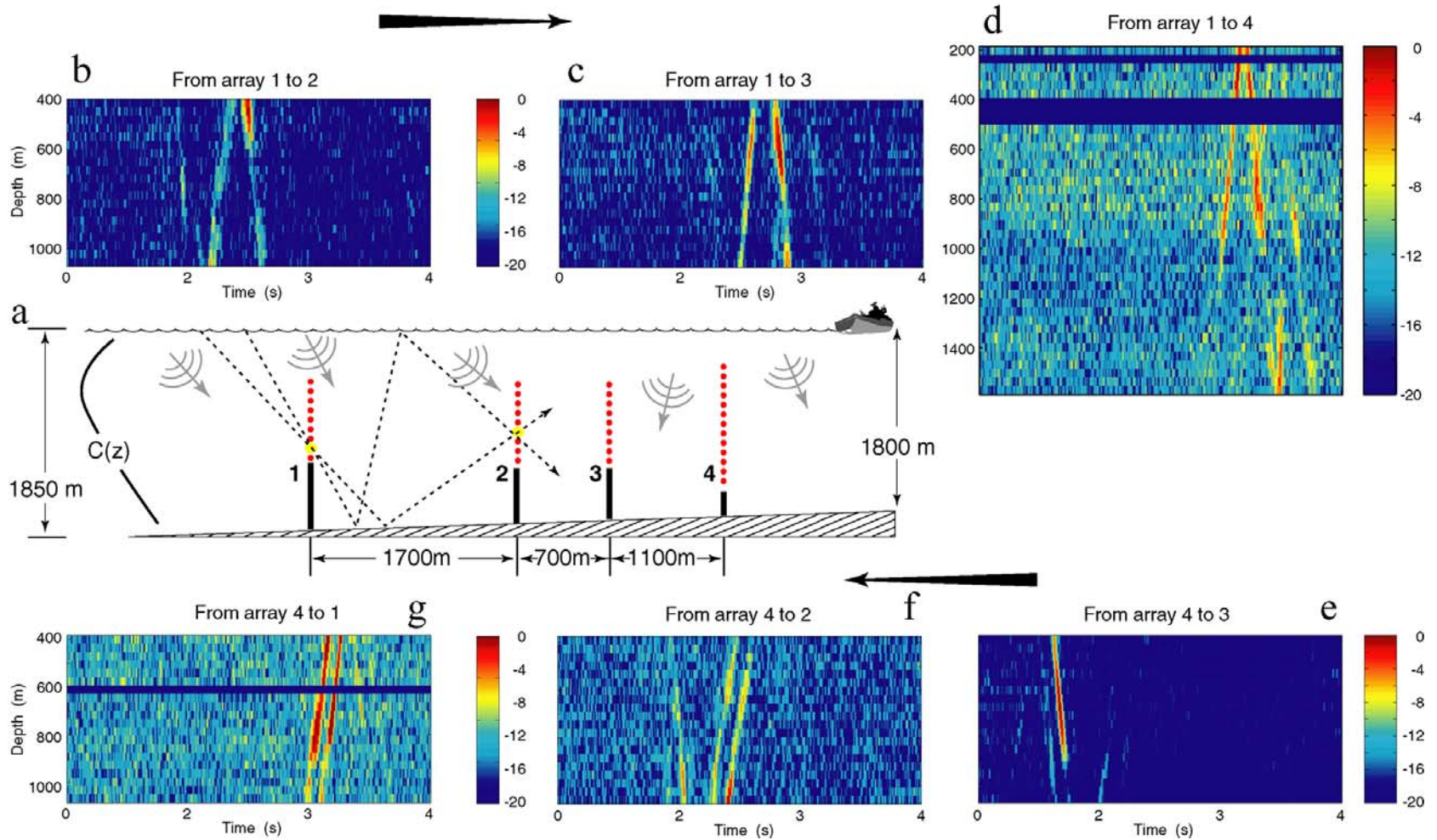
Coherent wavefronts yield an estimate of the Green's function between 1 and 2.

# Experimental results (70 – 130 Hz)

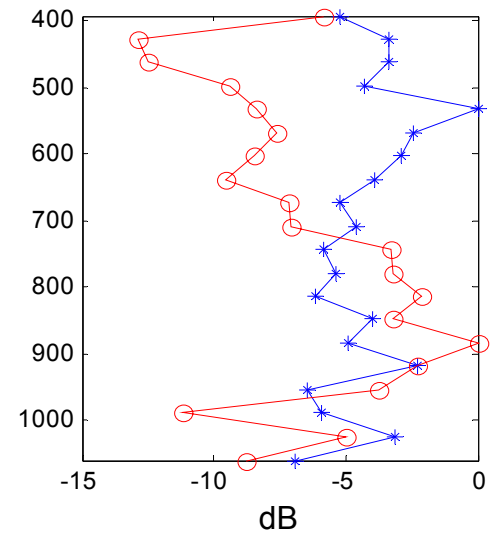
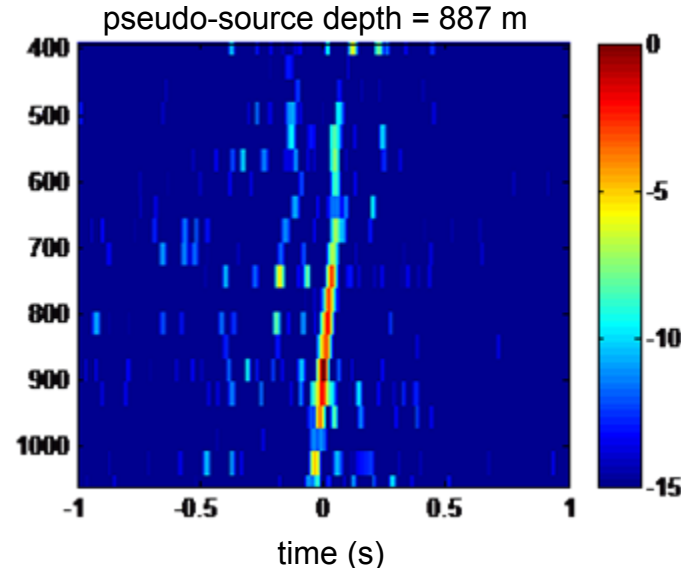
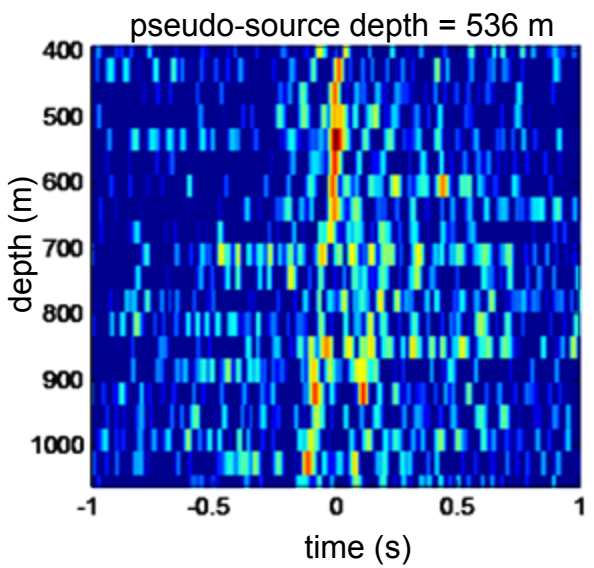
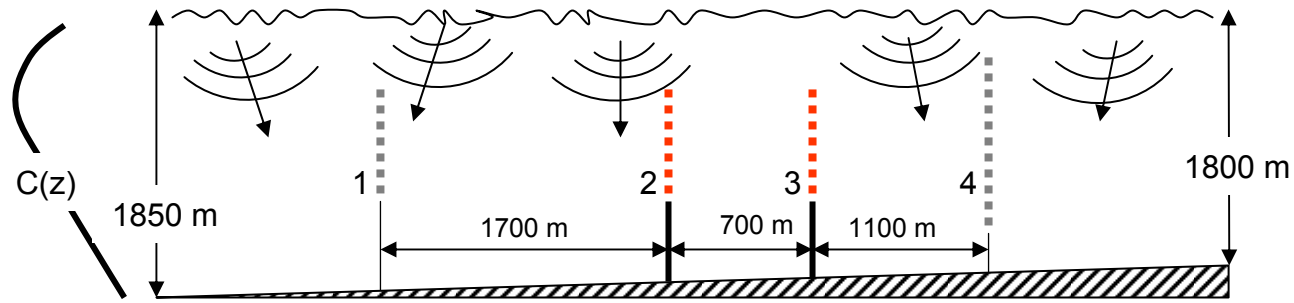
NPAL experiment  
(Worcester et al)



# Experimental results (70 – 130 Hz)

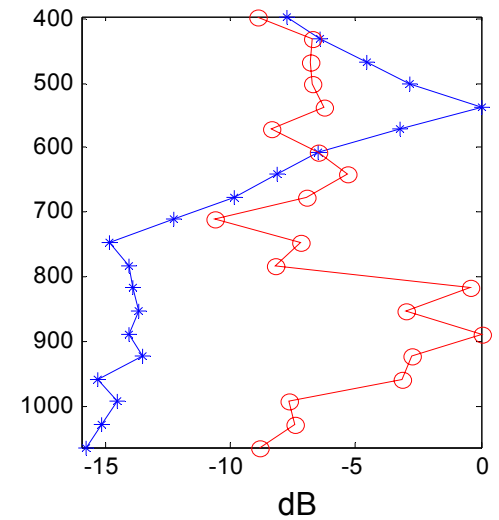
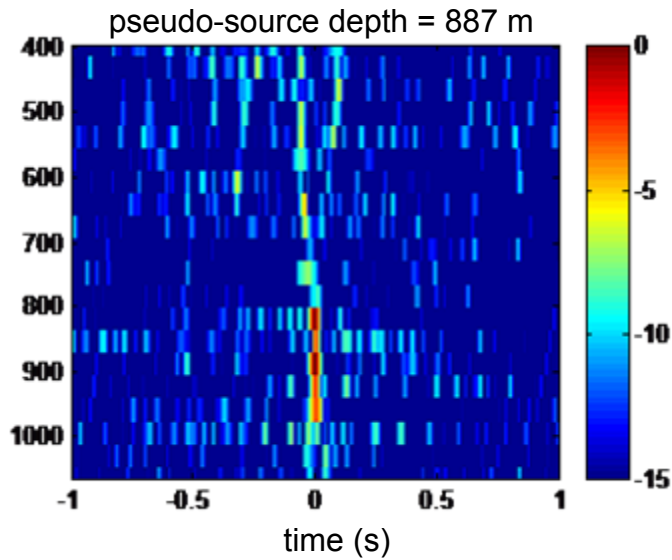
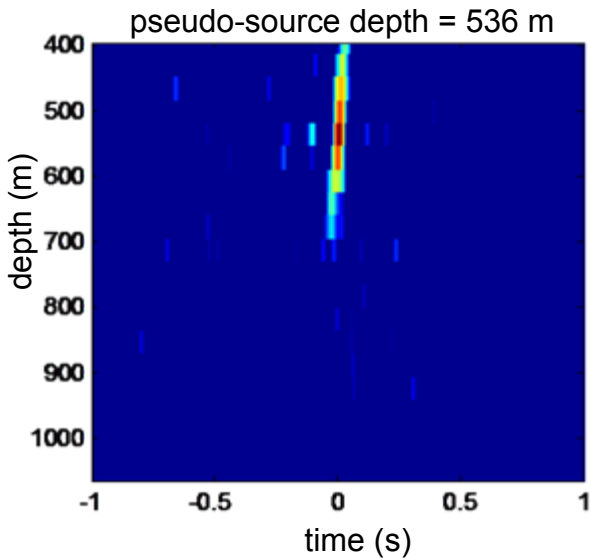
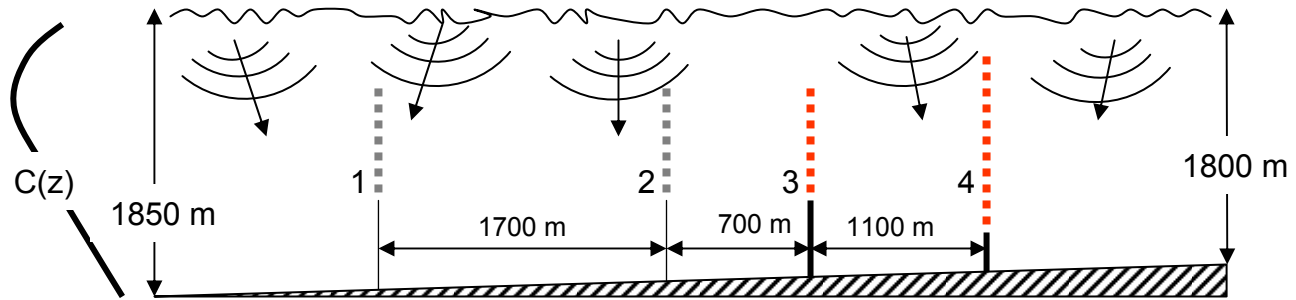


# Time reversal using noise as a probe source (70 – 130 Hz) [2->3]

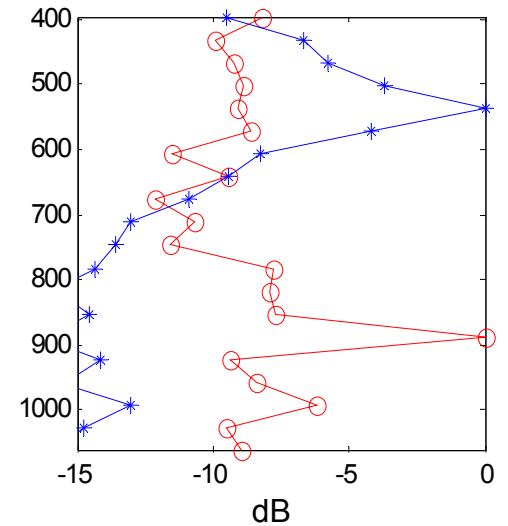
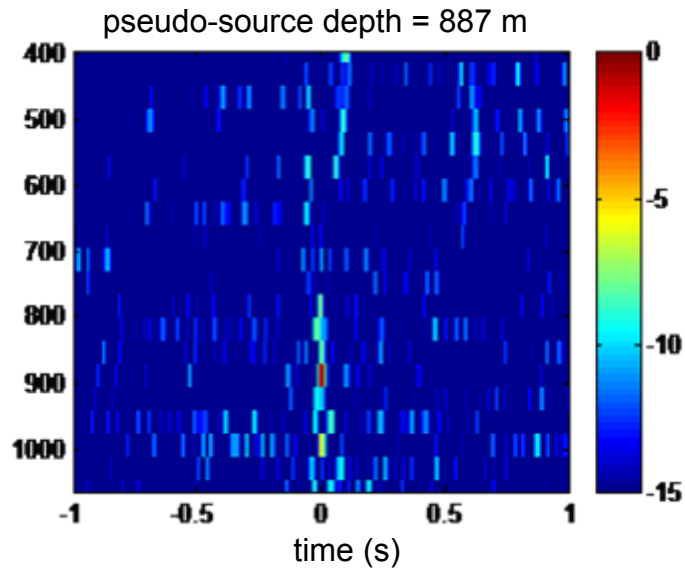
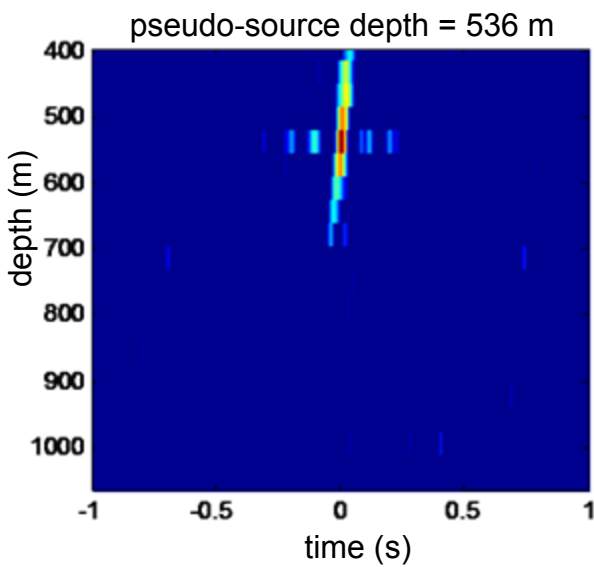
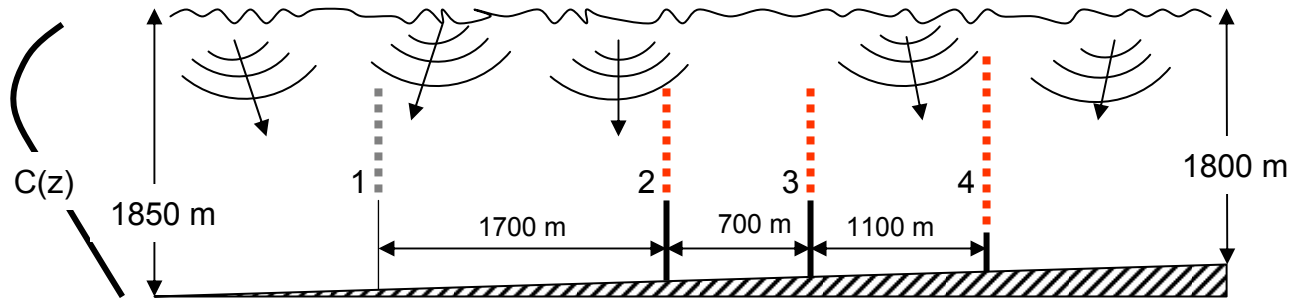




# Time reversal using noise as a probe source (70 – 130 Hz) [4->3]

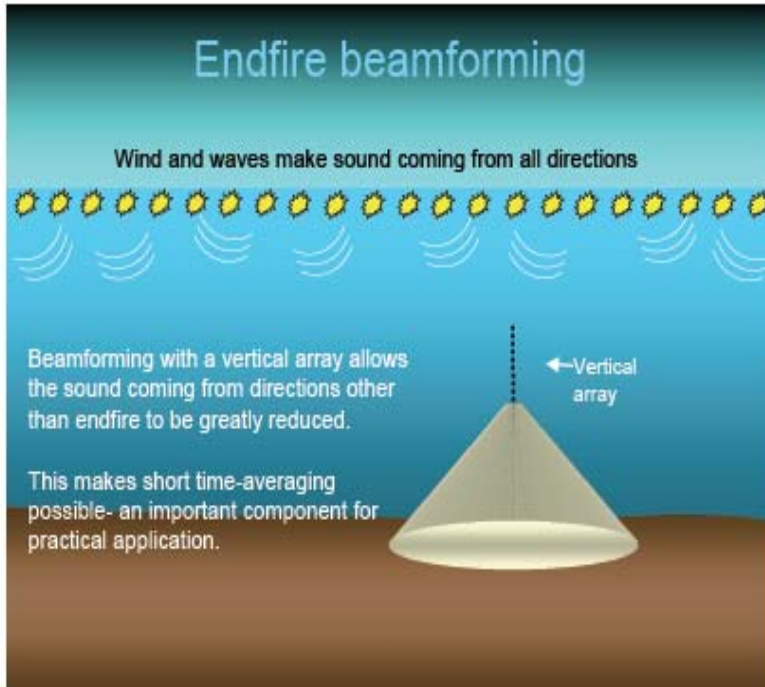


# Time reversal using noise as a probe source (70 – 130 Hz) [2&4->3]

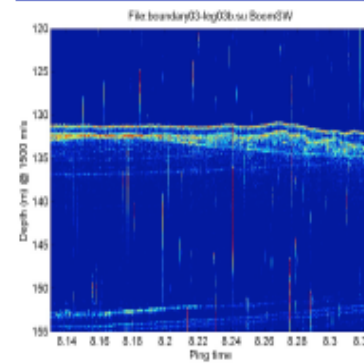


# VERY RECENT APPLICATION

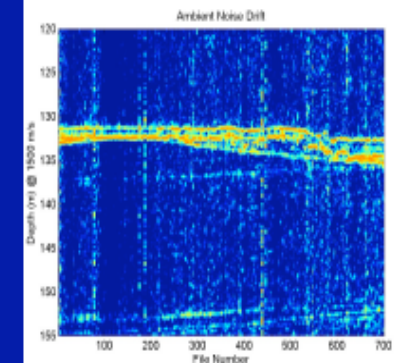
A passive fathometer and sub-bottom profiler using ambient noise



## Results: Drifting array (NURC Boundary2003 Experiment)

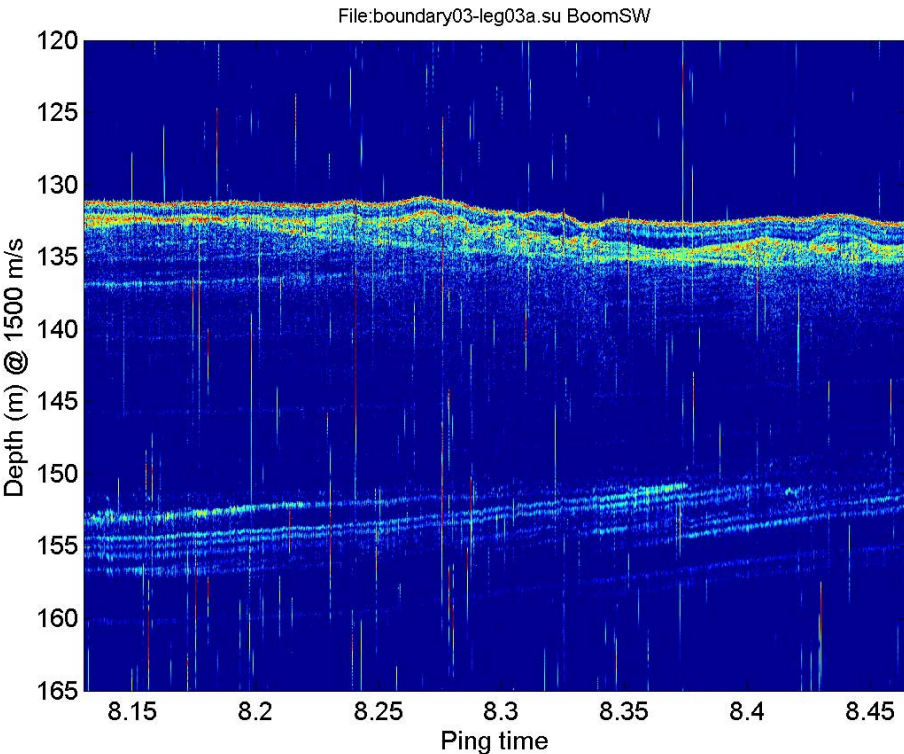


Sub-bottom survey with Uniboom system

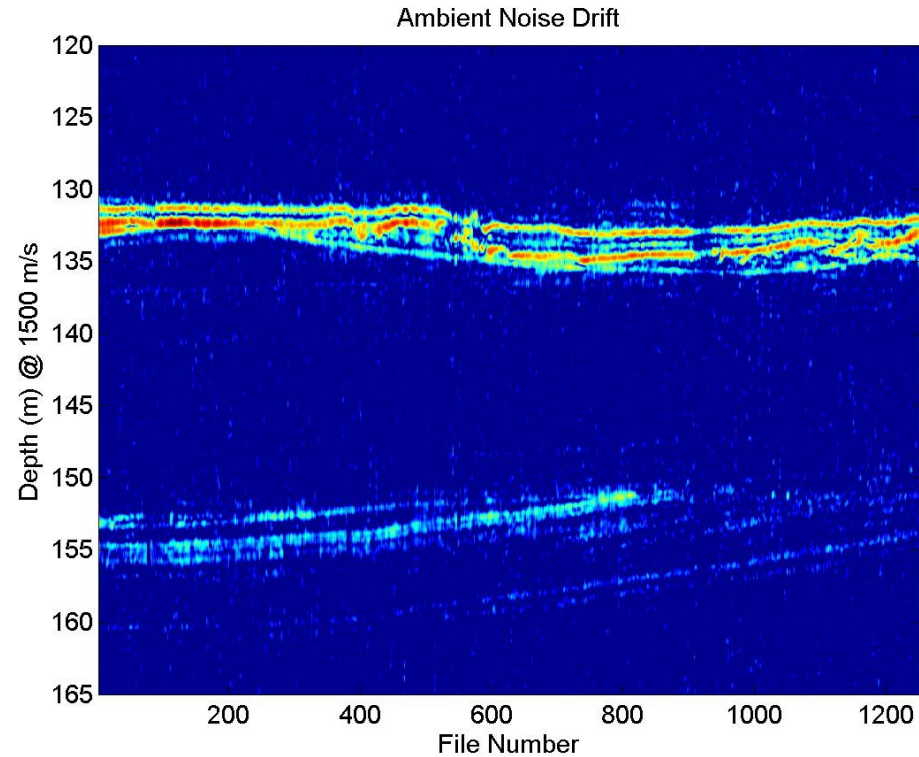


Sub-bottom survey using ambient noise

# Adaptive beamforming



Sub-bottom survey with Uniboom system



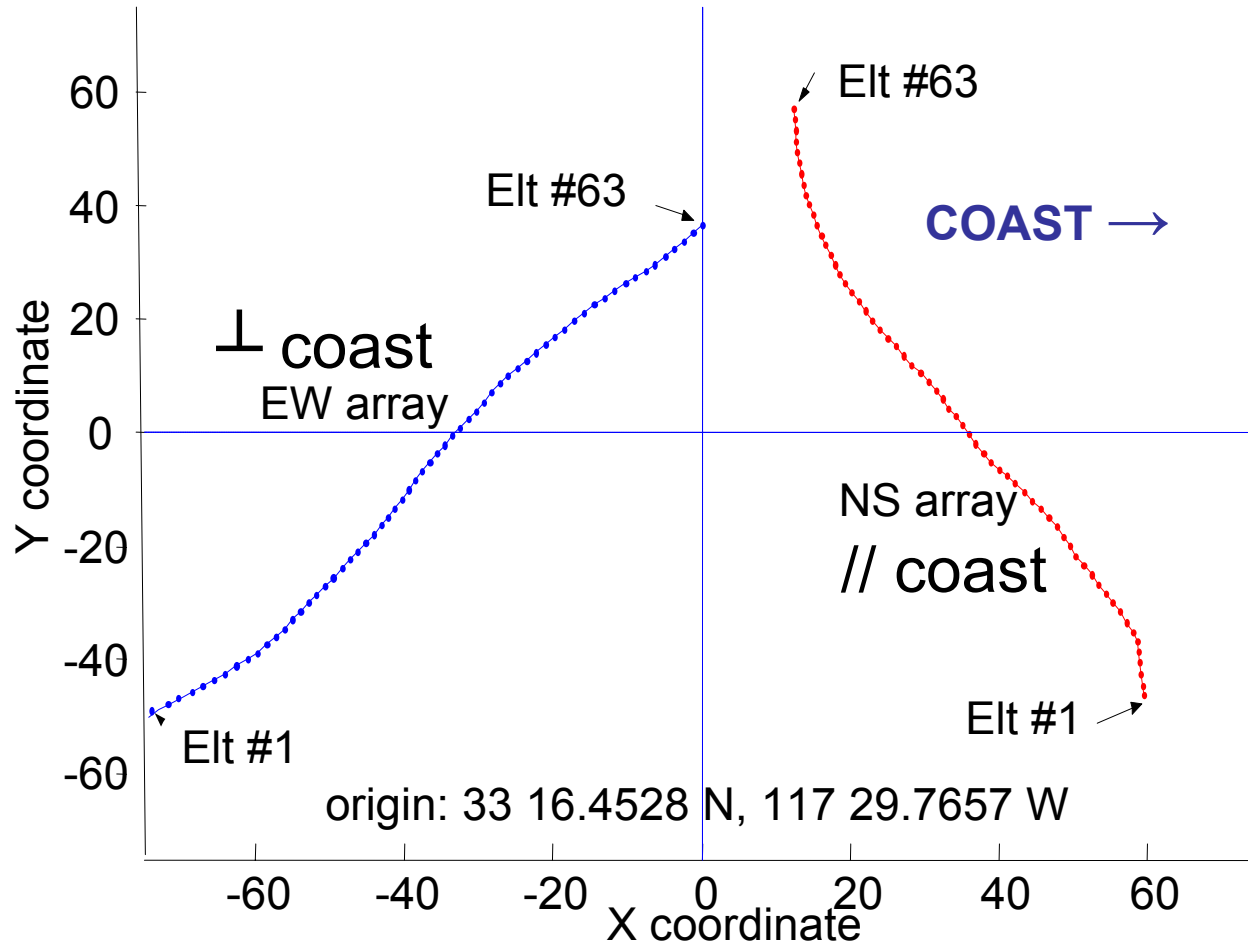
Sub-bottom survey using ambient noise and adaptive beamforming





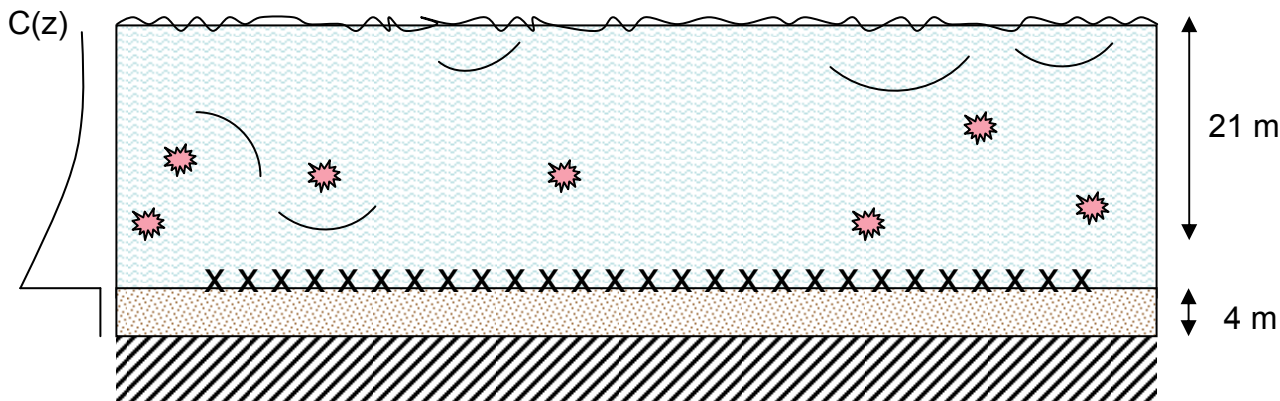
# Experimental Set-Up

- Adaptive Beach Monitoring experiment (ABM 95) Spring 95. South Calif.
- 2 bottom arrays were, 3.4km offshore,  $H = 21\text{m}$  of water
- 4m very fine sand sediment layer, high attenuation. Sandstone basement



- 64 elements  
( $D_{\max} = 1.875\text{m} \sim \lambda/2 @ 400\text{Hz}$ )
- Bandwidth: [2Hz-750Hz]. Flat response
- $F_s = 1500\text{ Hz}$ .
- 2 weeks continuous recordings of ambient noise

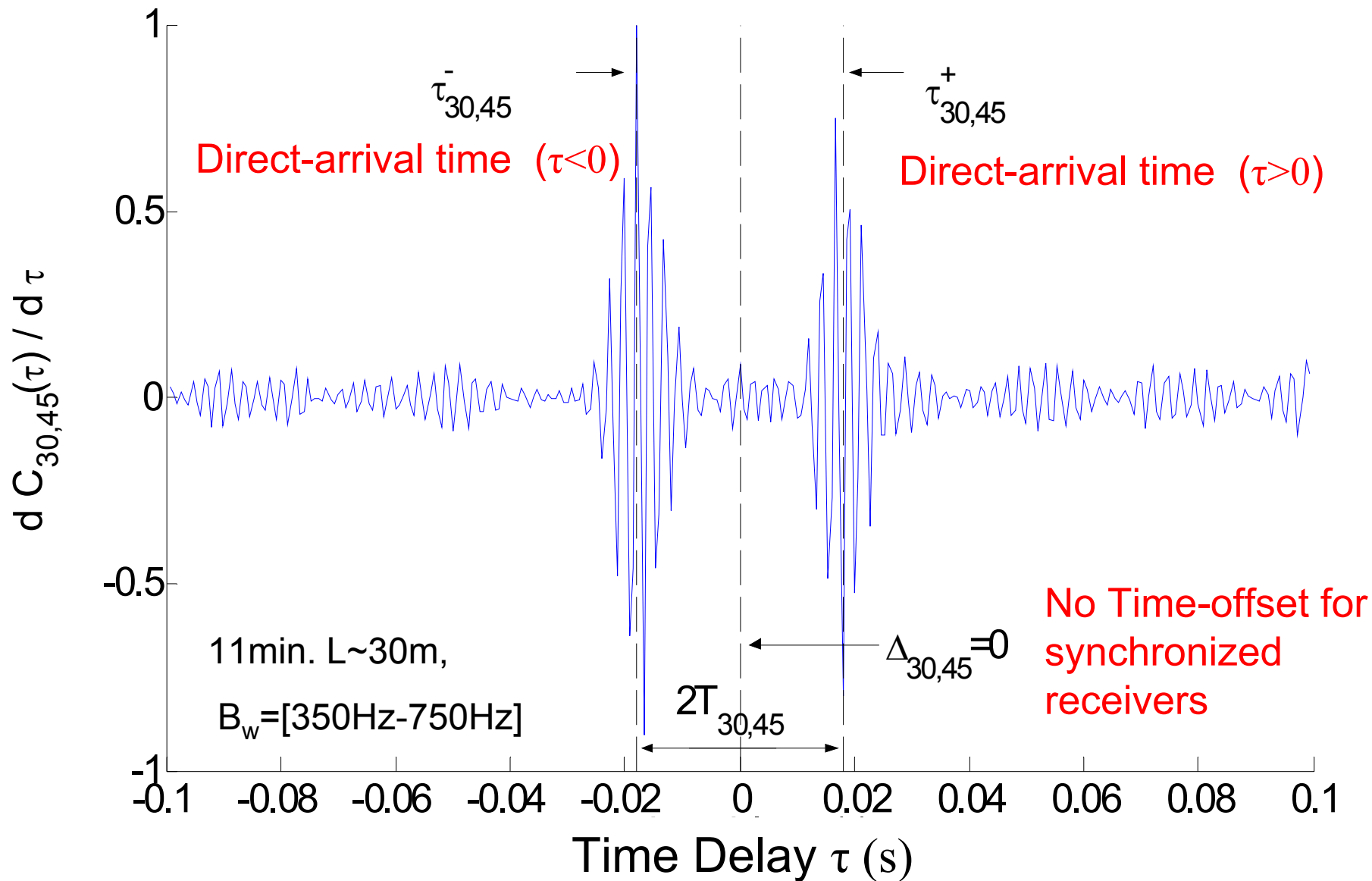
# Motivation



Determine environmental characteristics from cross-correlation of ambient-noise recordings along a horizontal array.

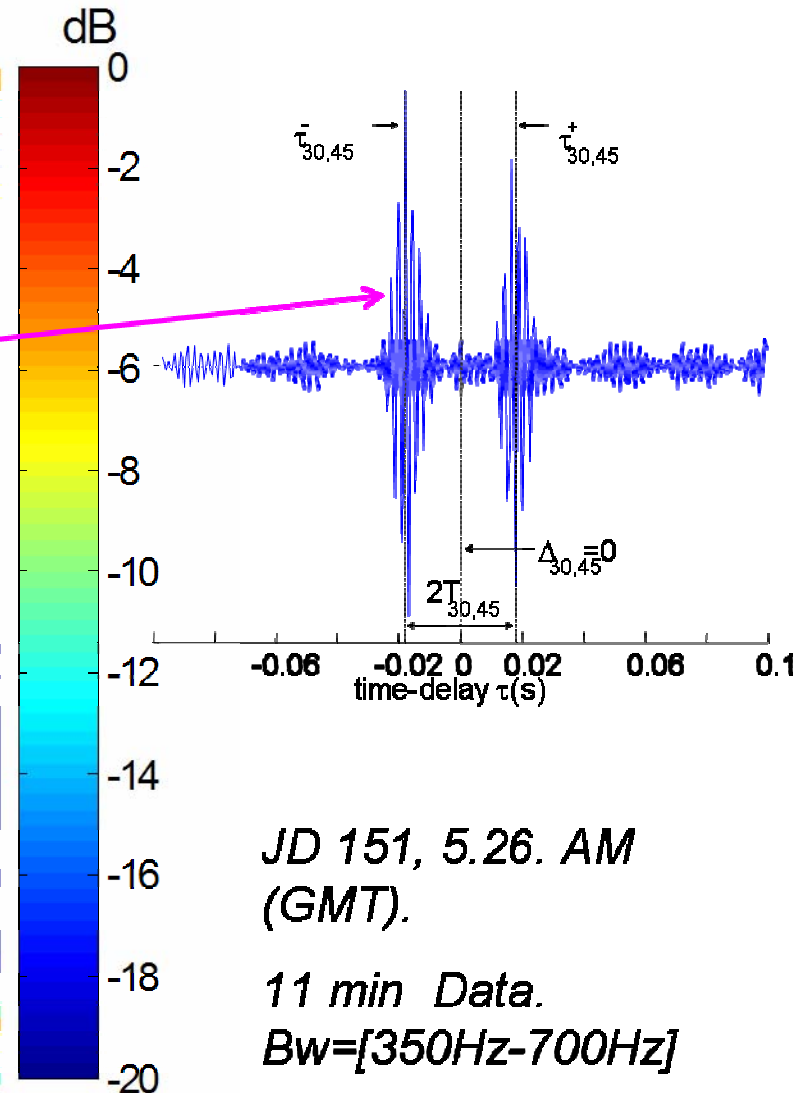
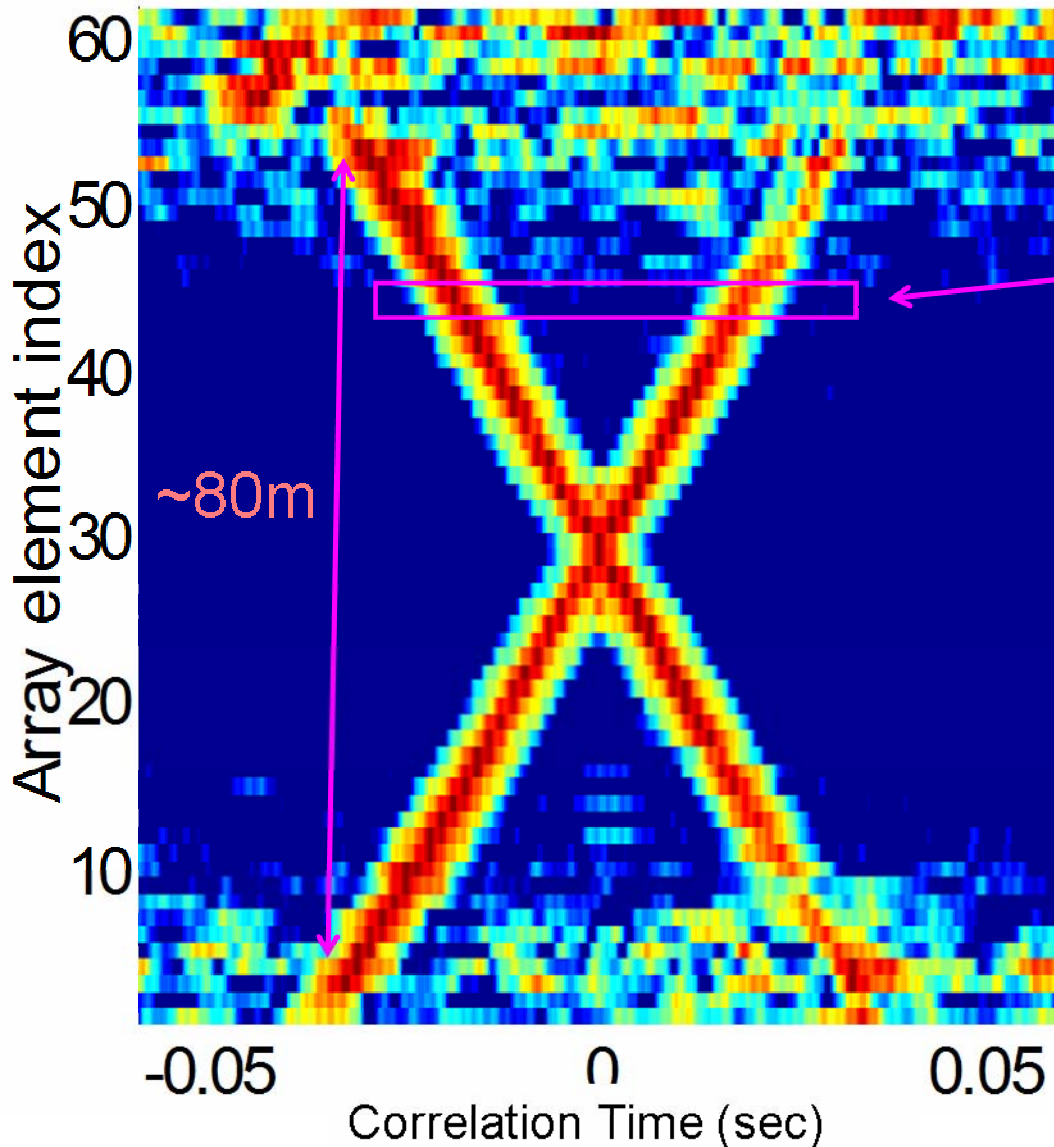
# Ambient-noise NCF

Time-derivative of the NCF. NS array, Elt 30-45. Symmetry w.r.t to time origin



# Horizontal Coherent Wavefronts

Reference array element: #30



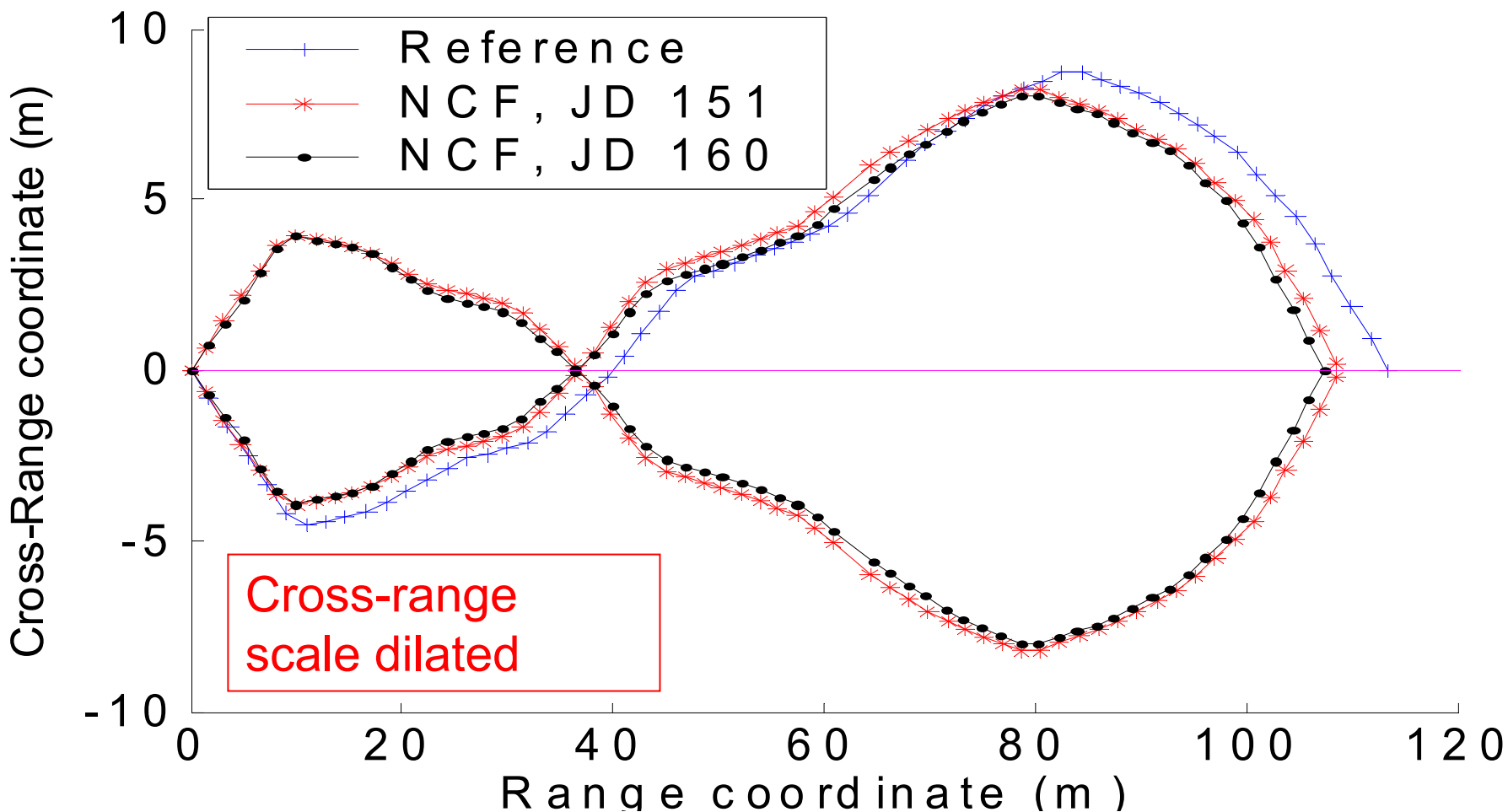
*JD 151, 5.26. AM  
(GMT).*

*11 min Data.*

*Bw=[350Hz-700Hz]*

# Array Element Self-Localization

- Non linear Least Square Inversion ( $2D+c_0$ ) for AEL.
- A-priori information:  $D_{max}=1.875m$ . Minimize array curvature.



*JD 151  $c_0=1490m/s$ , RMS Error~0.40m*

*JD 160  $c_0=1485m/s$ , RMS Error~0.43m*

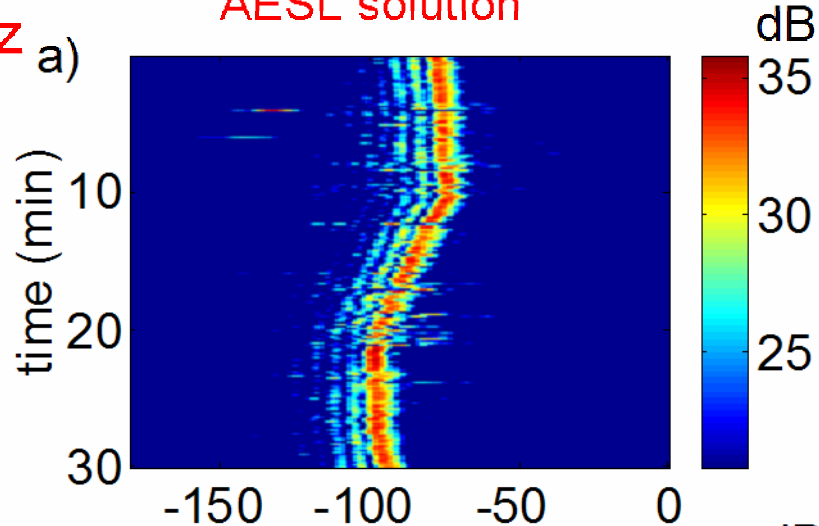
# Towed Source Beamforming

AEL from JD 151  $c_0=1490\text{m/s}$ . Plane wave beamforming. CSDM with 5sec FFT.

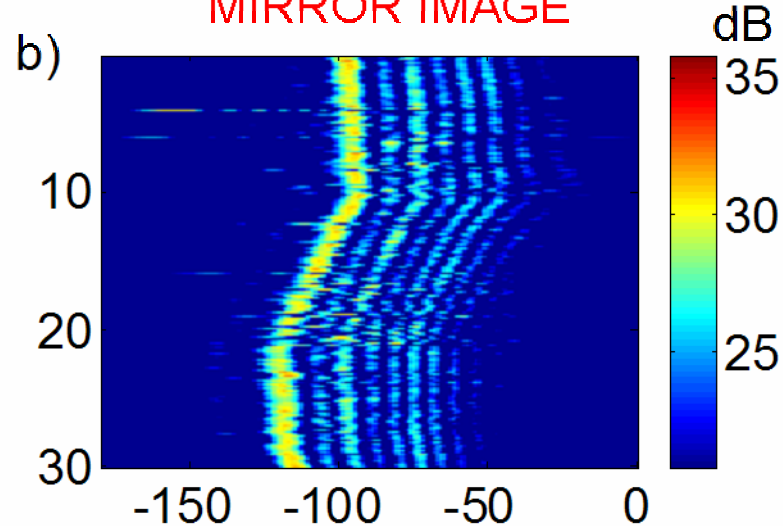
$N=63$  Elts;  $20\log N \sim 36\text{dB}$

145Hz

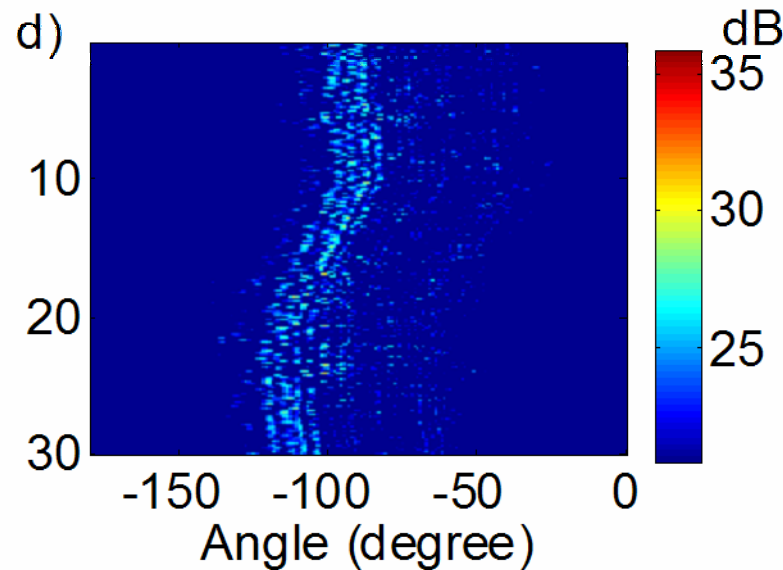
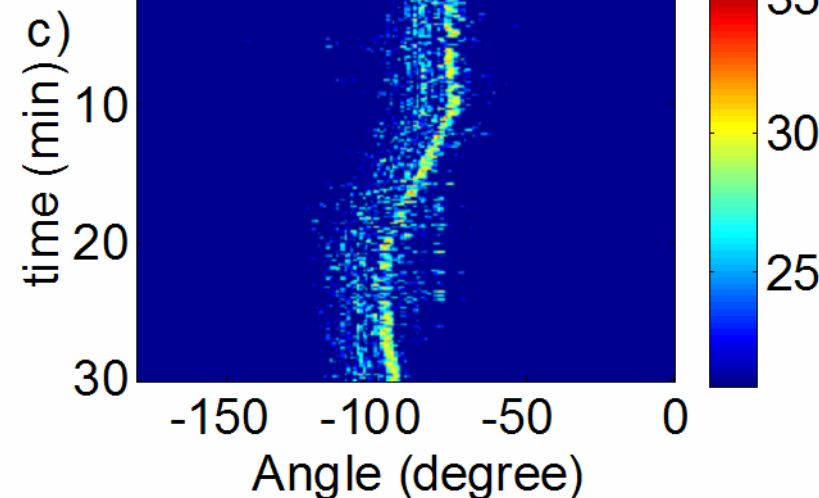
AESL solution



MIRROR IMAGE

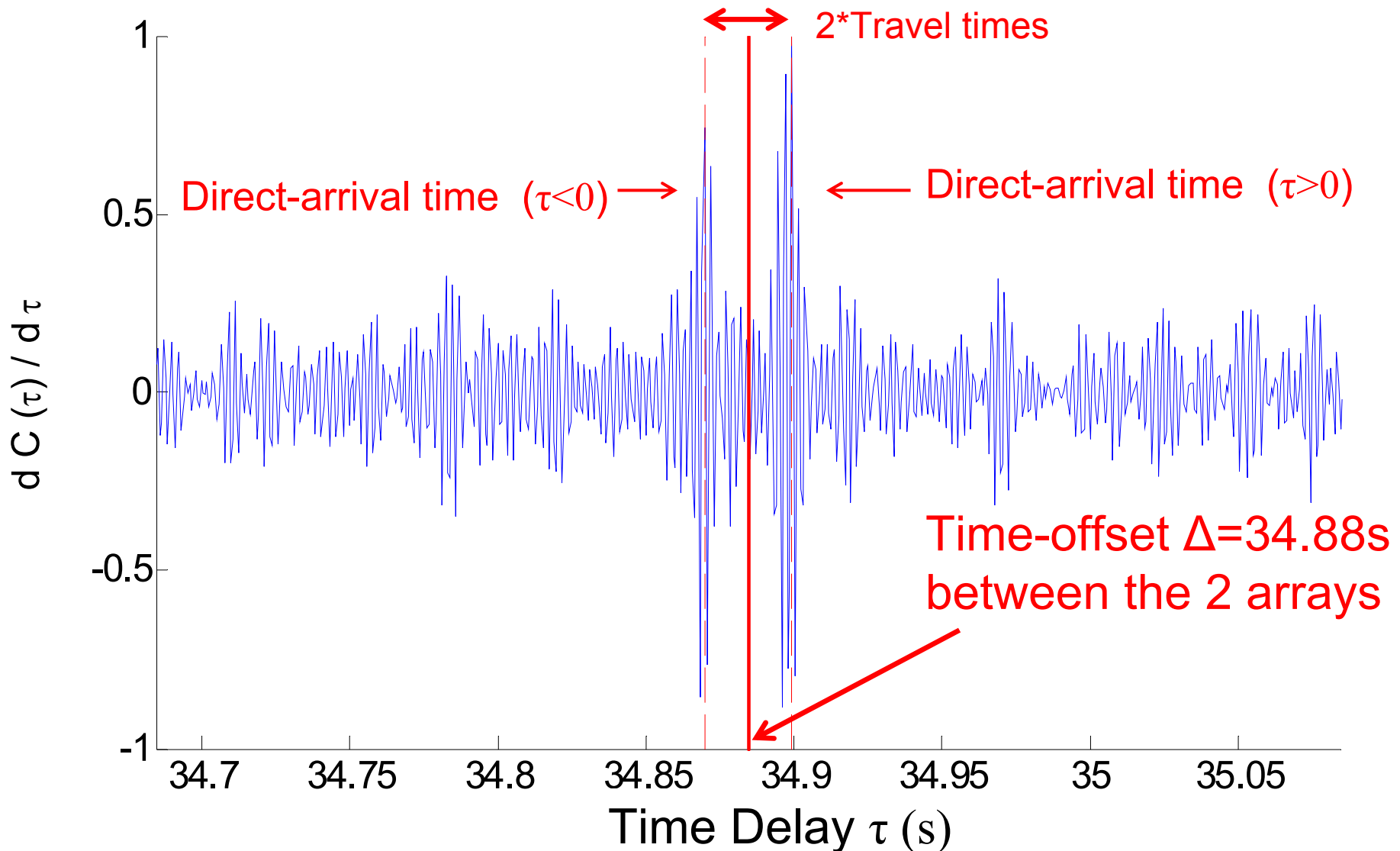


370Hz



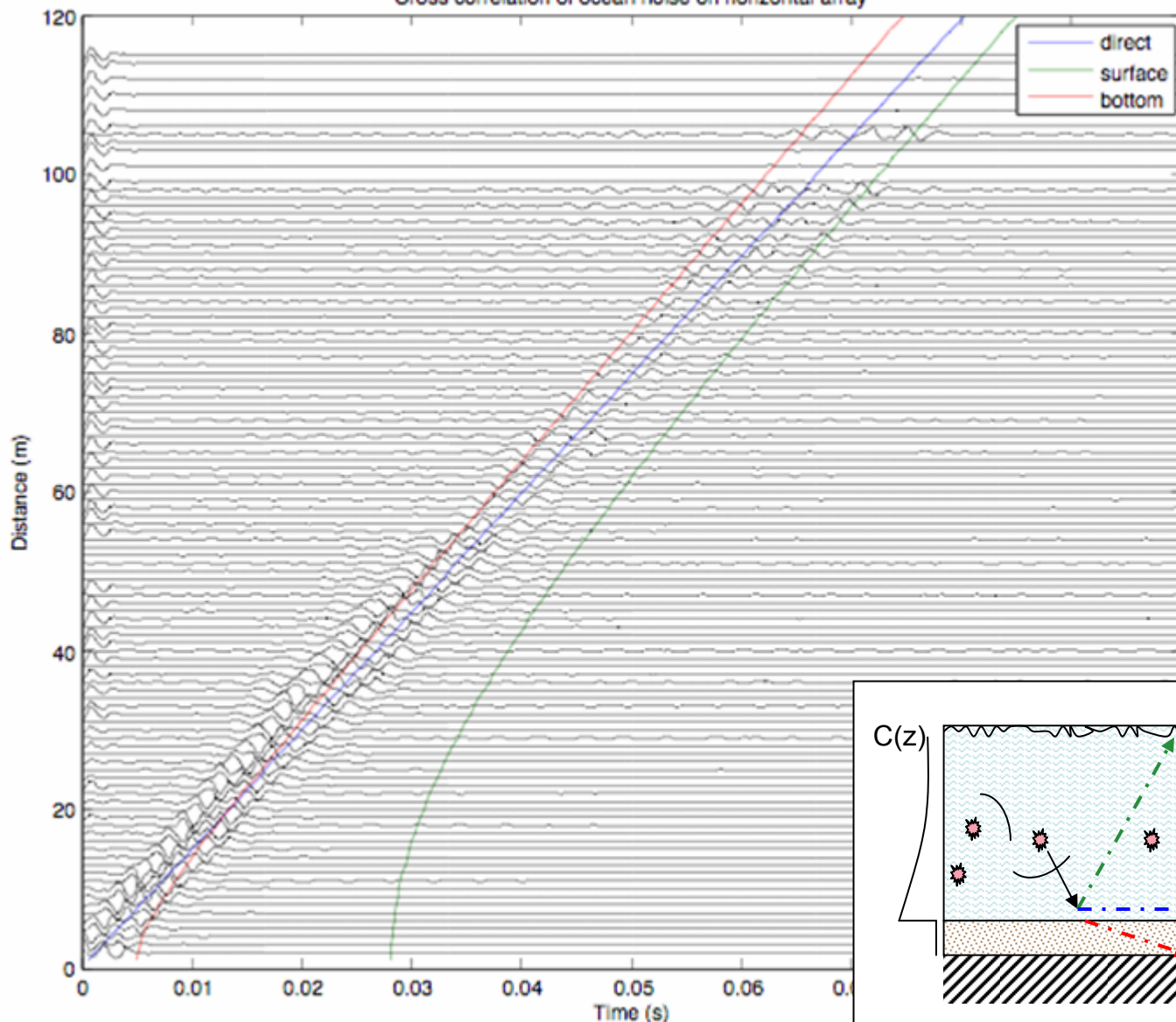
# Array Element Self-Synchronization

Time-derivative of the NCF. NS & EW array, Elt 63 → Time-shifted origin



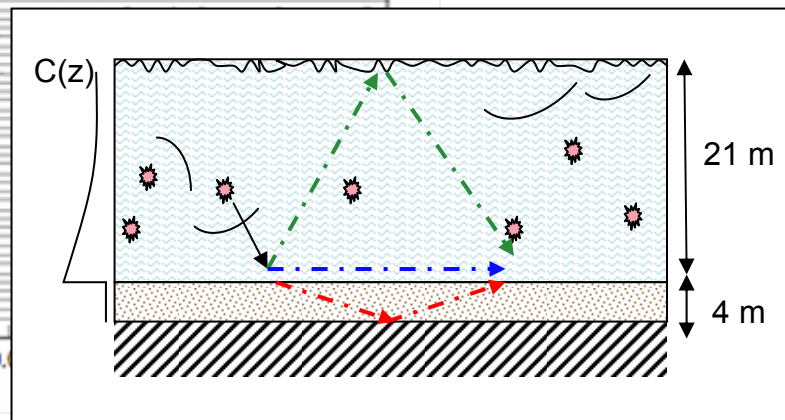
# NCF traces (Data)

Cross correlation of ocean noise on horizontal array



-NCF stacked by separation distance of receivers.

- Colored lines represent simplified isovelocity description of propagation paths.

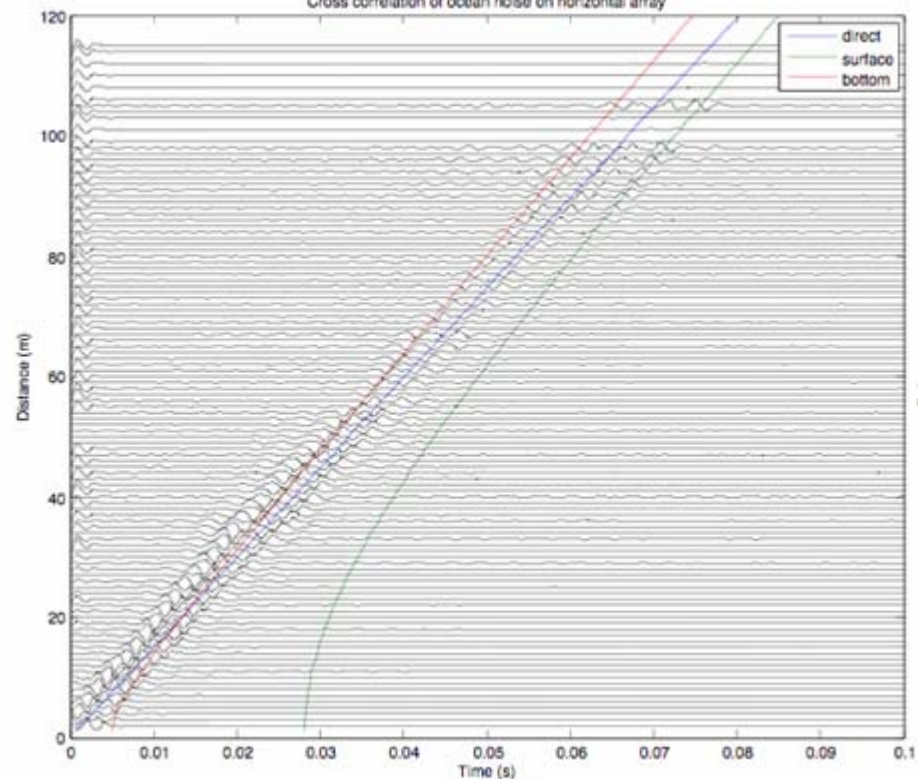




# Compare NCF & Simulation

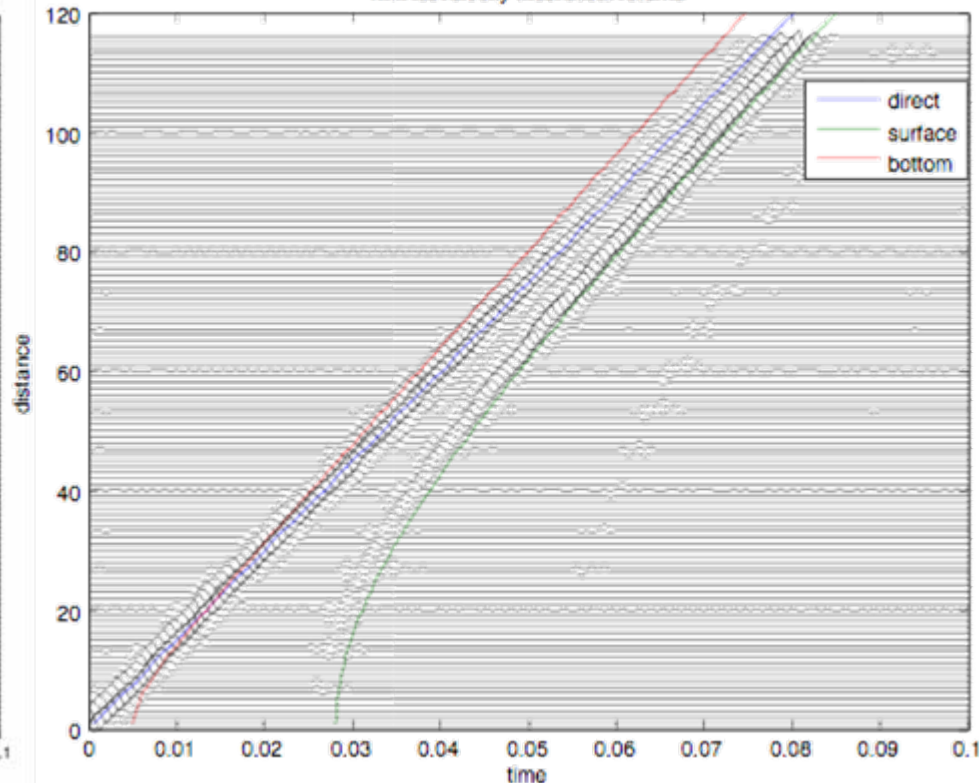
## NCF of data

Cross correlation of ocean noise on horizontal array

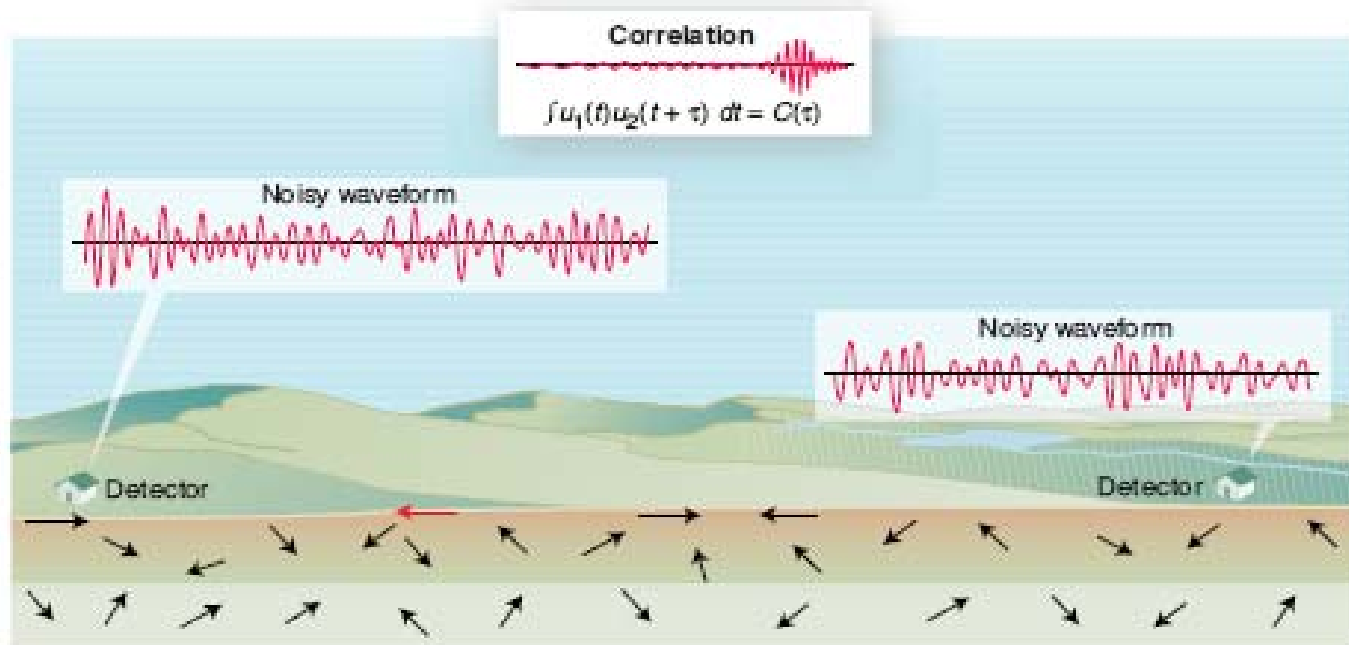


## Spectral simulation of Green's fnc

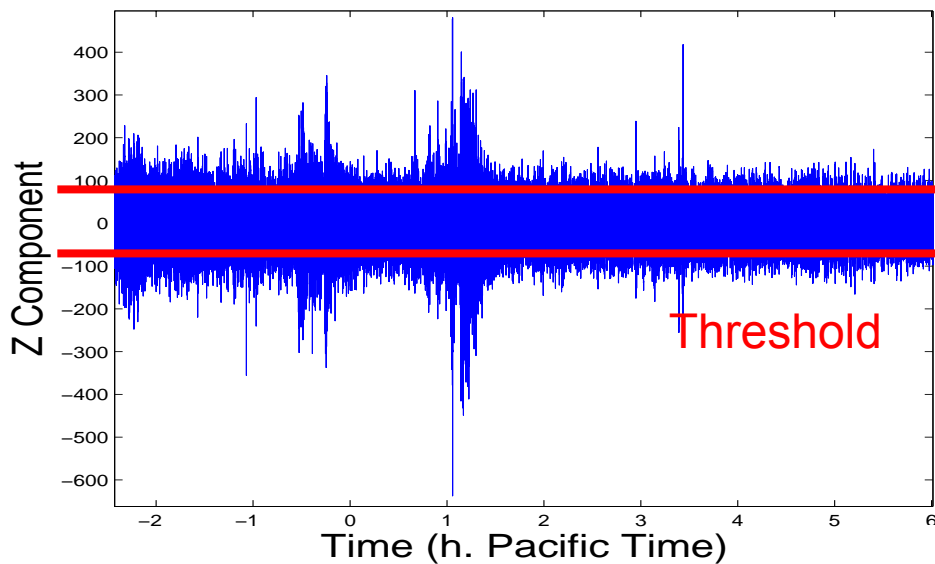
Spectral simulation  
with isovelocity theoretical returns



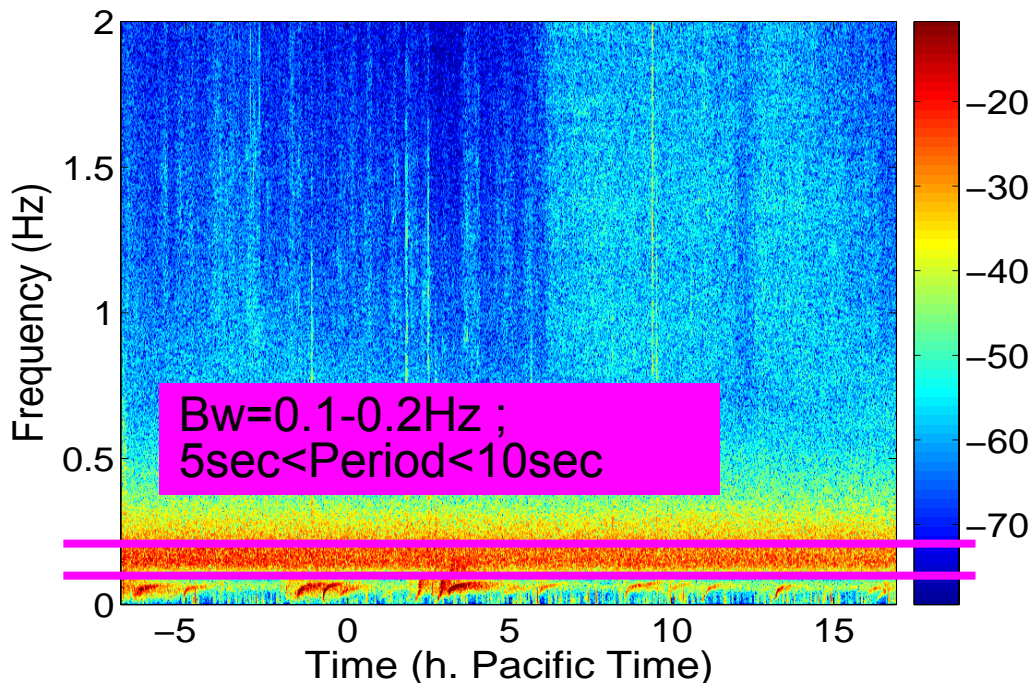
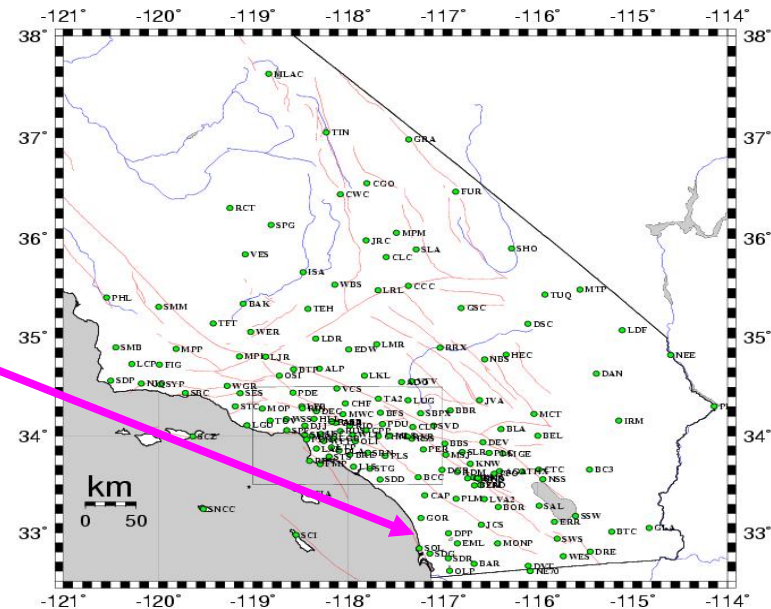
# High resolution surface wave tomography from ocean microseisms in Southern California



# Vertical Component



# Station Location



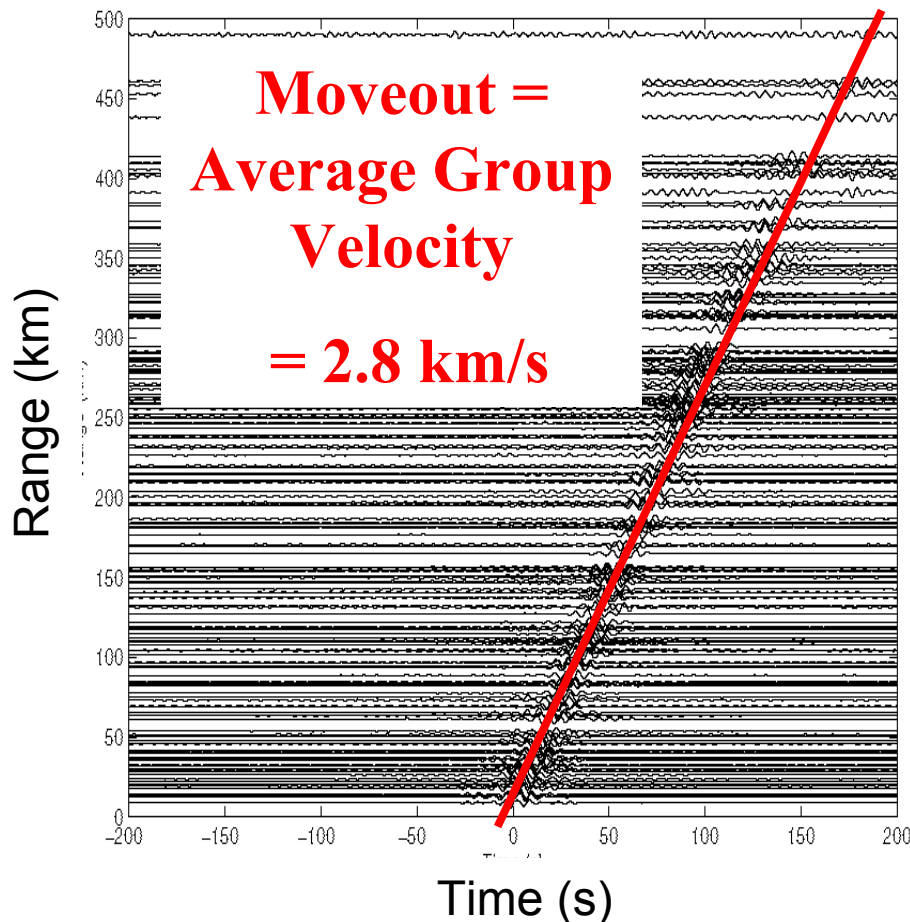
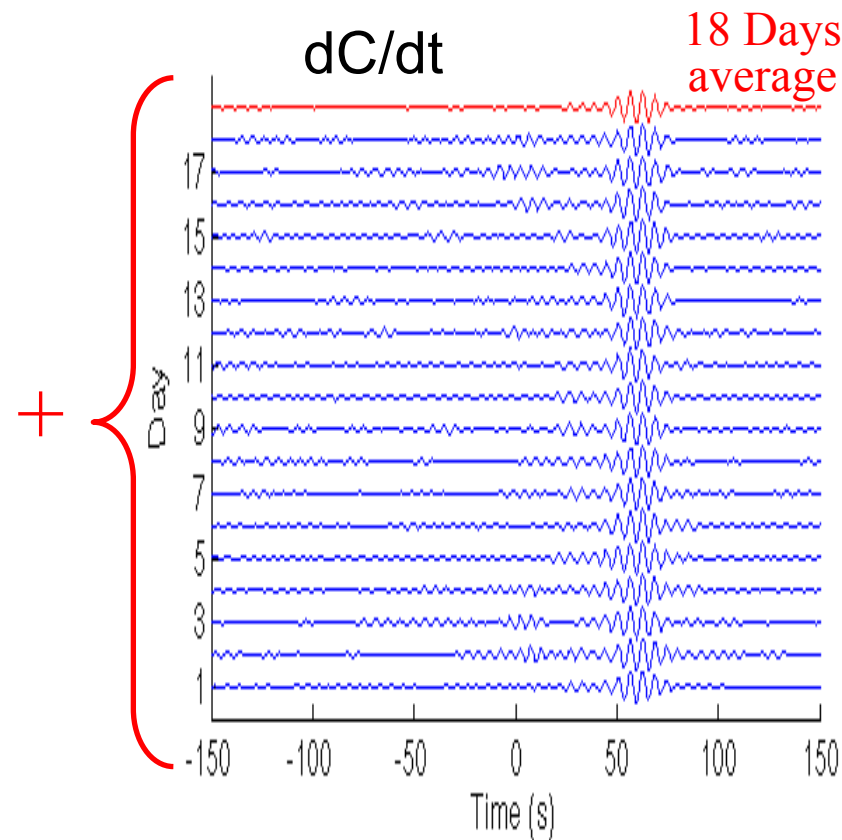
Southern California Seismic Network  
(150 Stations)  
Broadband data.  $F_s=10\text{Hz}$ .  
18 Days of continuous data

Ocean microseisms  
propagate mainly as  
Rayleigh waves

# Emergence rate of coherent waveforms

$B_w = [0.1-0.2\text{Hz}]$ , Z-comp. R=150 km

“Passive Shot Gather”.



$$SNR = \max(\text{Trace}) / \text{std}(\text{Trace})_{\tau > 150s}$$

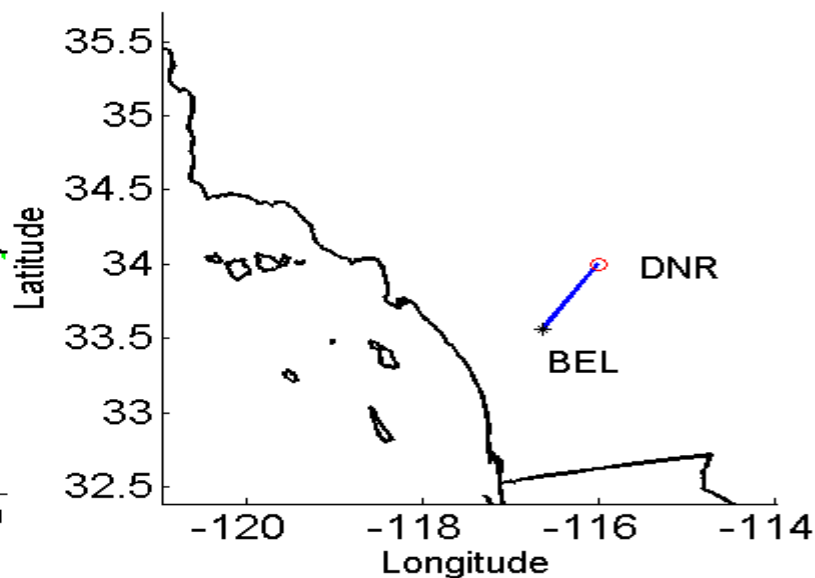
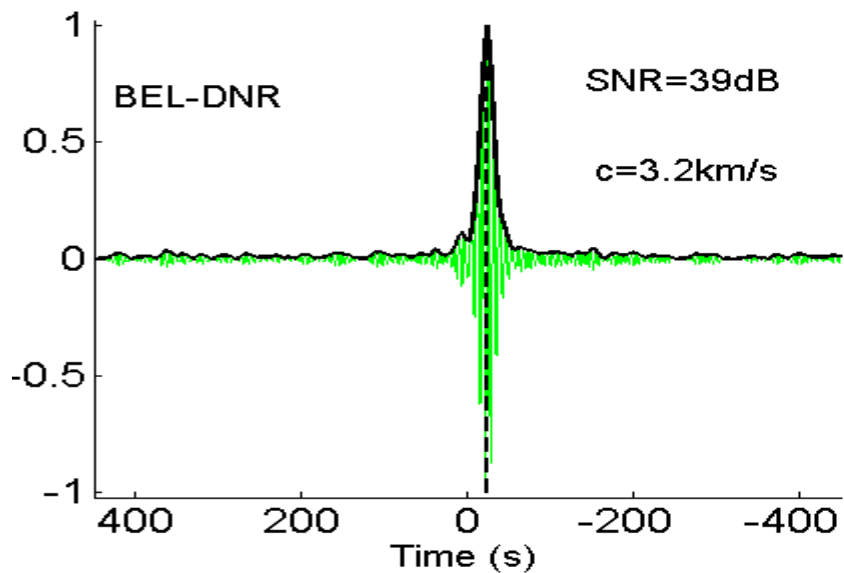
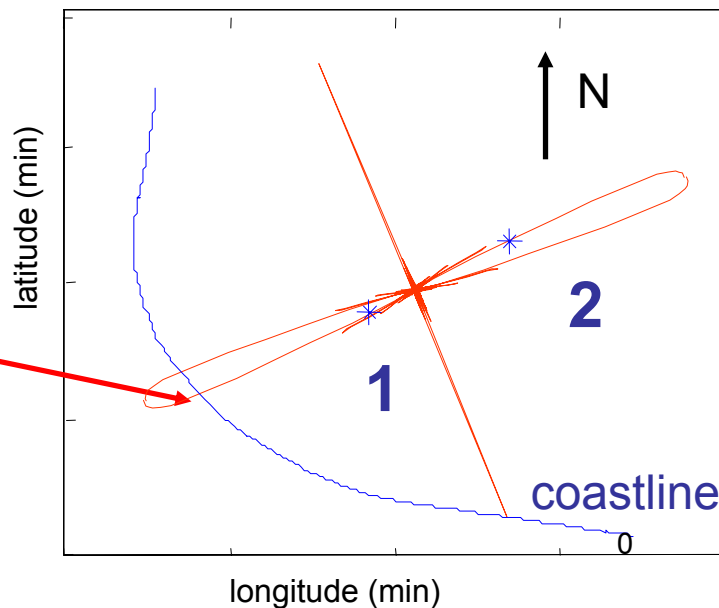
$$SNR \propto \sqrt{T_r B_\omega}$$

Station Pair (oriented  $0^\circ$  -  $21^\circ$  North)  
i.e. perpendicular to the coastline  
(directionality of the ocean microseisms)

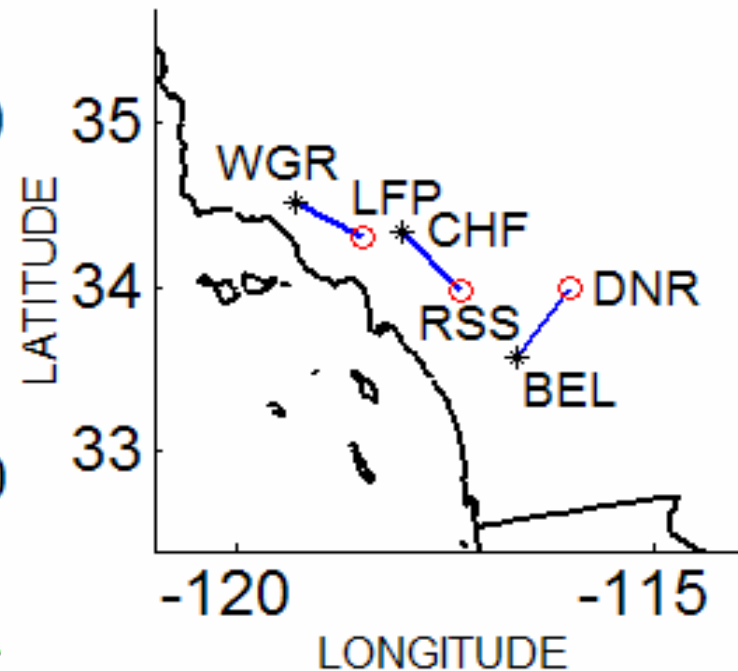
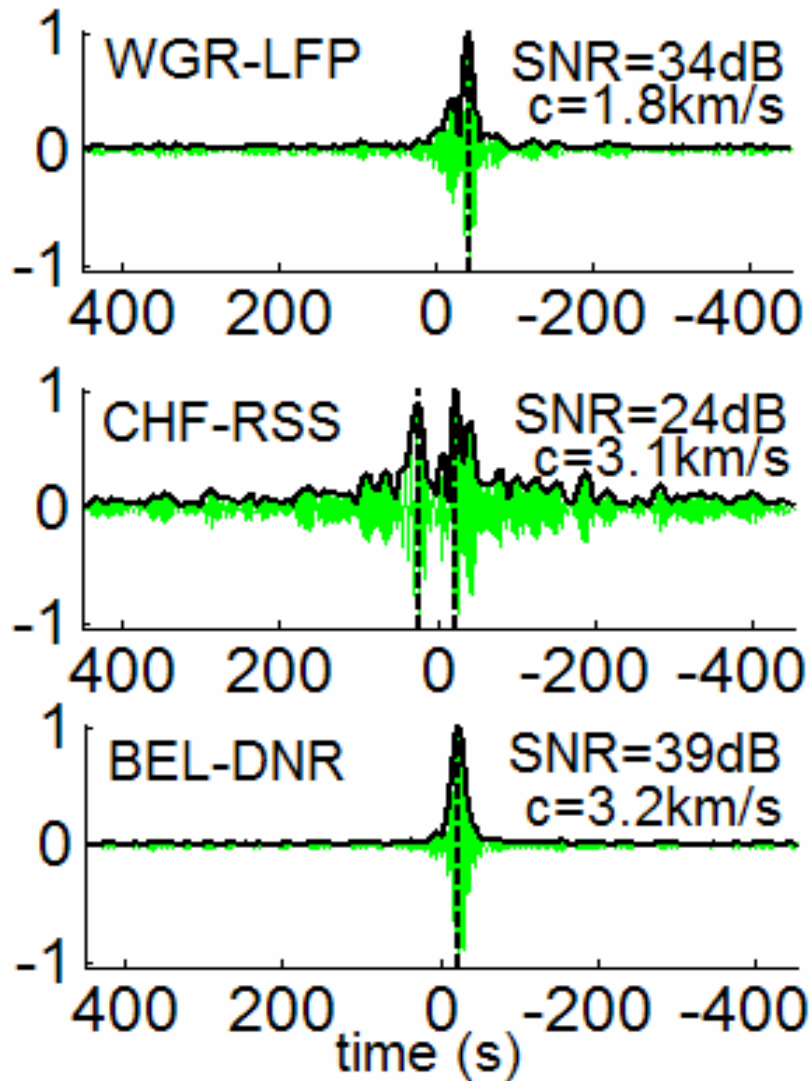


# Spatial resolution & stationary phase

Stationary  
Phase  
Lobes



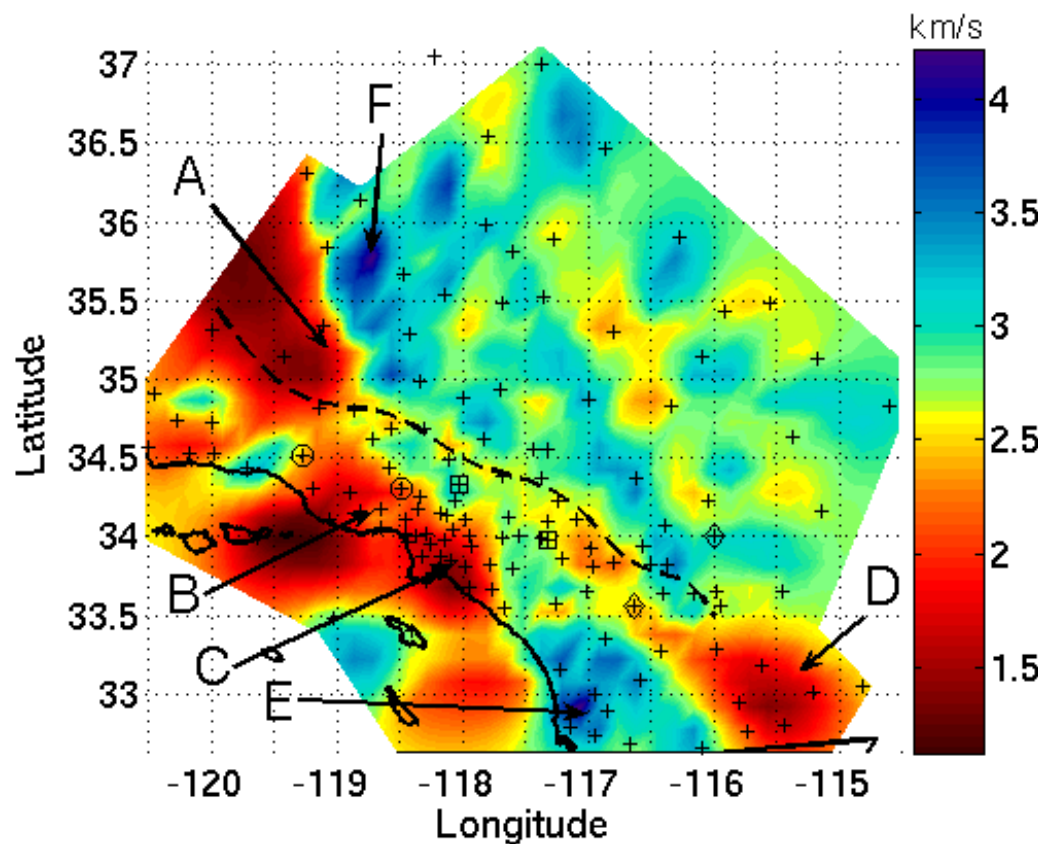
# 2D variations of the Rayleigh wave





# Surface wave Tomography 7.5s

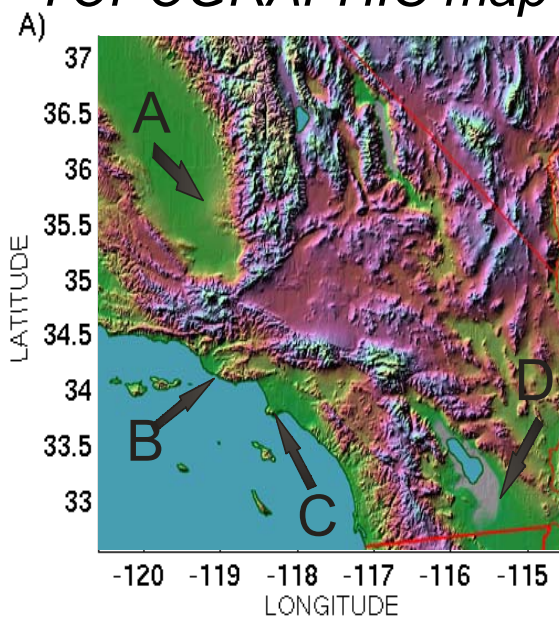
## TOMOGRAPHIC map



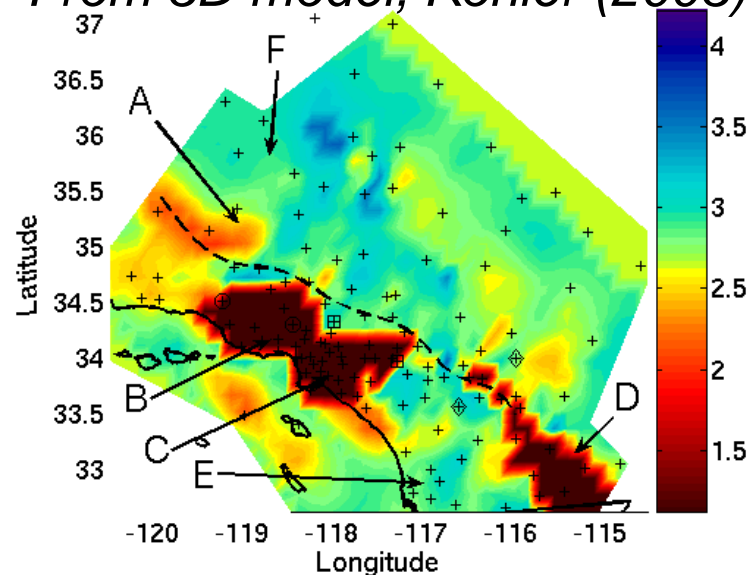
Grid 13\*16km, uniform *a priori* velocity:  
 $c=2.8\text{km/s}$ .  $\sigma_T=2\text{s}$ ,  $\sigma_c=0.15\text{km/s}$ (errors)

A: San Joaquin, B: Ventura, C: L.A., D: Salton Sea,  
E: Peninsular ranges, F: Sierra Nevada

## TOPOGRAPHIC map



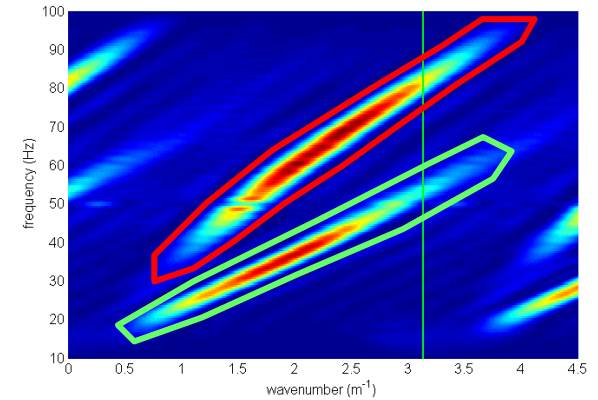
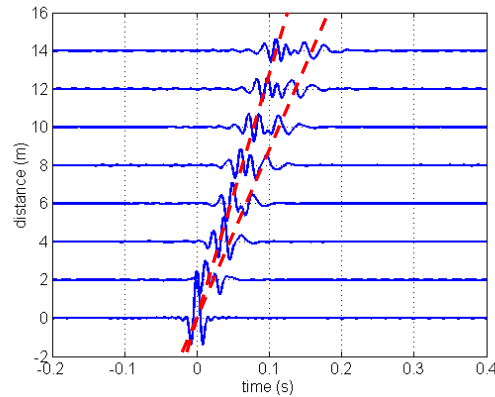
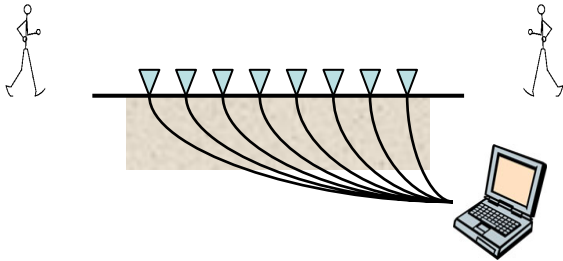
From 3D model, Kohler (2003)



# Small scale geophysics inversion -in your backyard

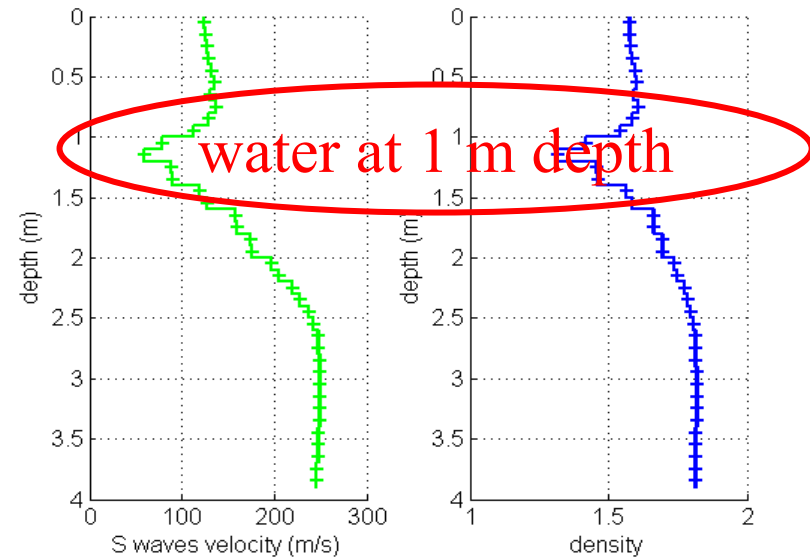
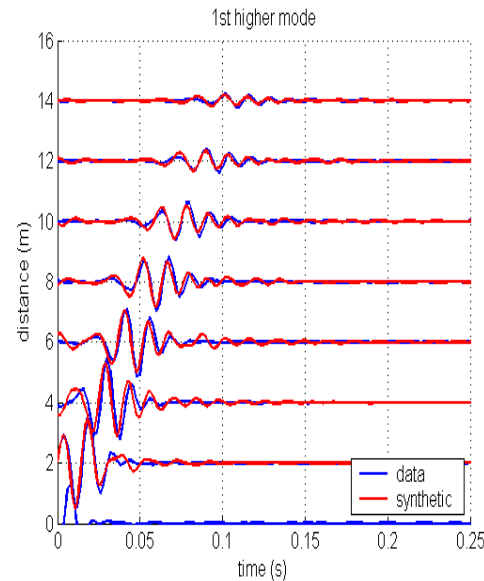
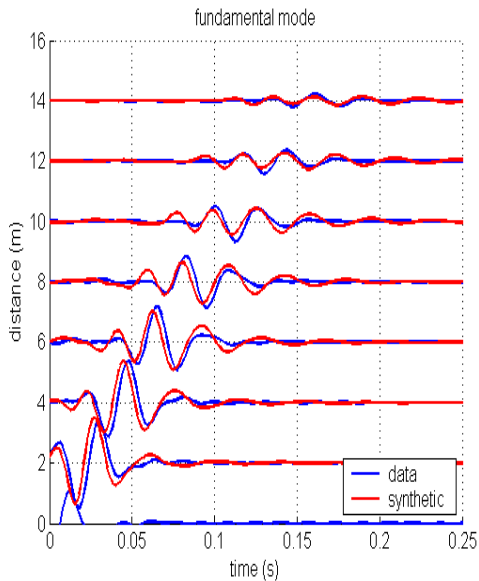
## 1-Cross-Correlations

## 2-FK transform



## 3-Extracting Rayleigh modes

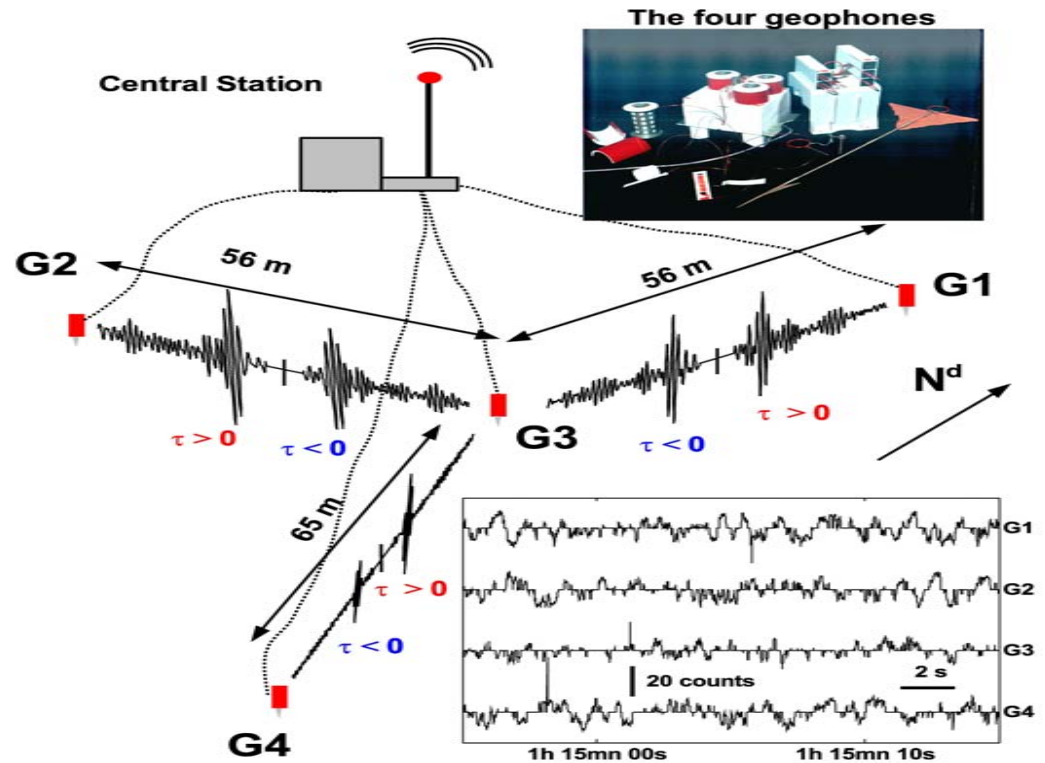
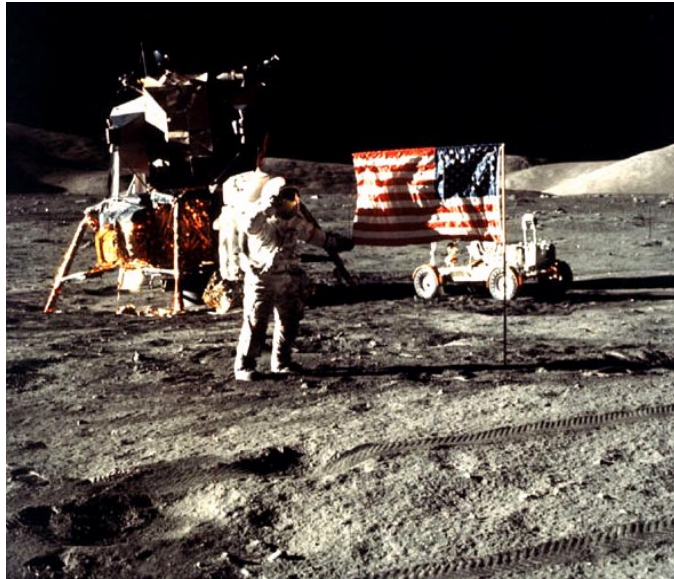
## 4-Inversion from dispersion curves





# Cross-correlations of seismic noise *on the Moon* !

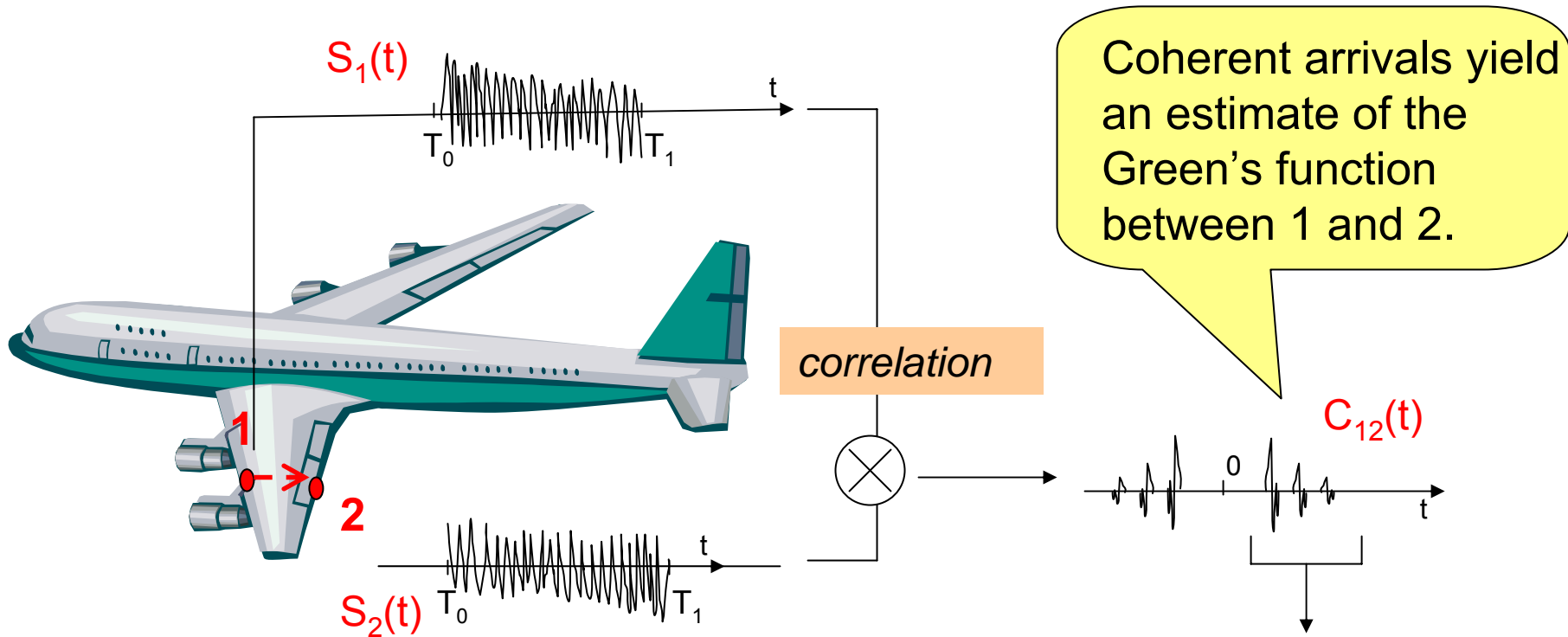
Seismic noise origin:  
Thermal Cracks  
( $-170^{\circ}\text{C}$  /  $+110^{\circ}\text{C}$ )



Larose et al., *Geophys. Res. Lett.*, **32** (2005)

*Applications exist in a wide range of environments and frequency bands because the physics driving this noise cross-correlation process remains similar.*

# Using Structural Noise

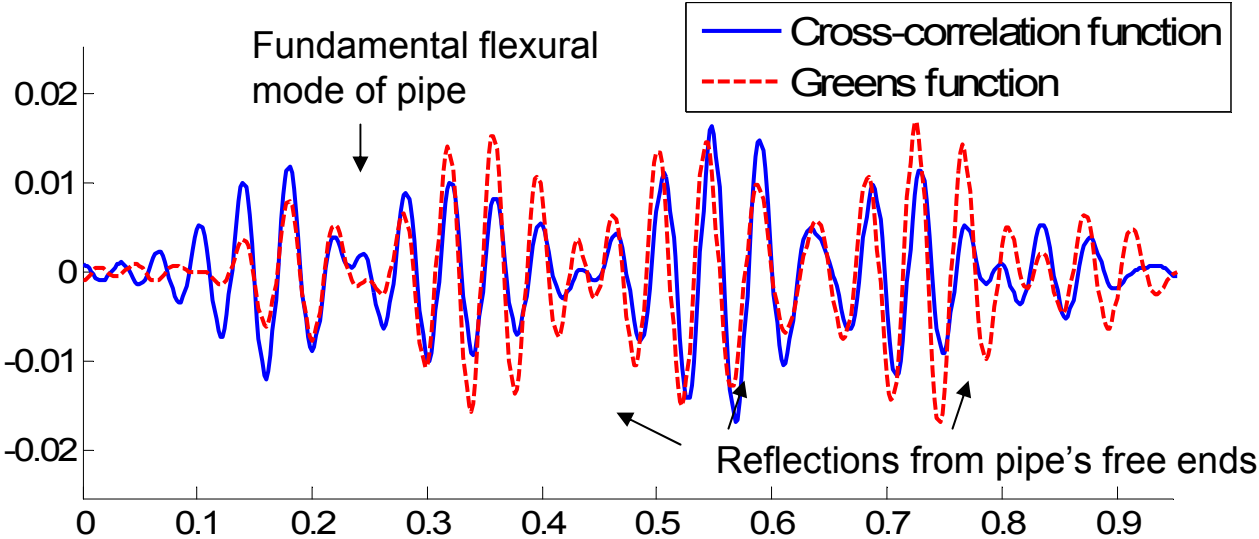
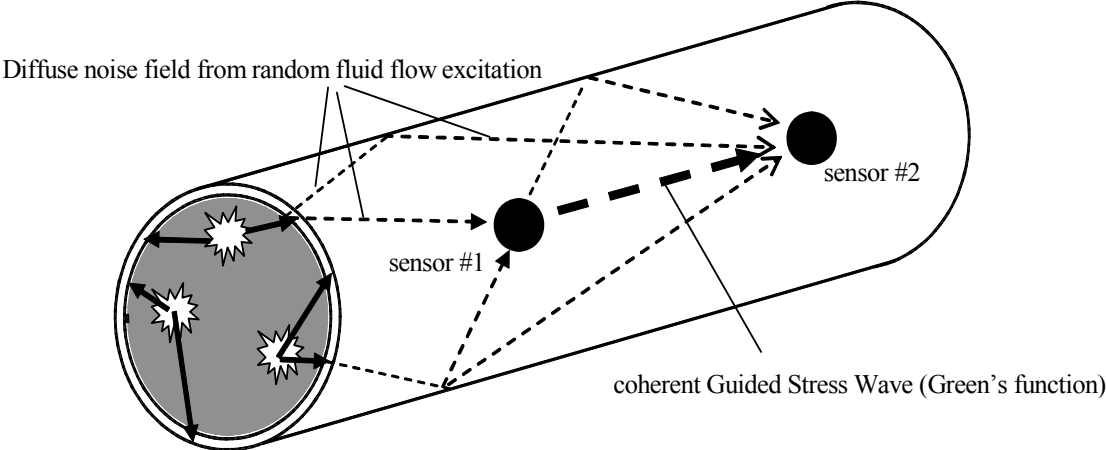


Acoustic and vibration waves that propagate through the locations of two receivers coherently average and dominate the cross-correlation function of the receiver pair for long time records.

# Test Concept: Structural Health Monitoring of Pipeline Bases on Flow-Induced Vibrations



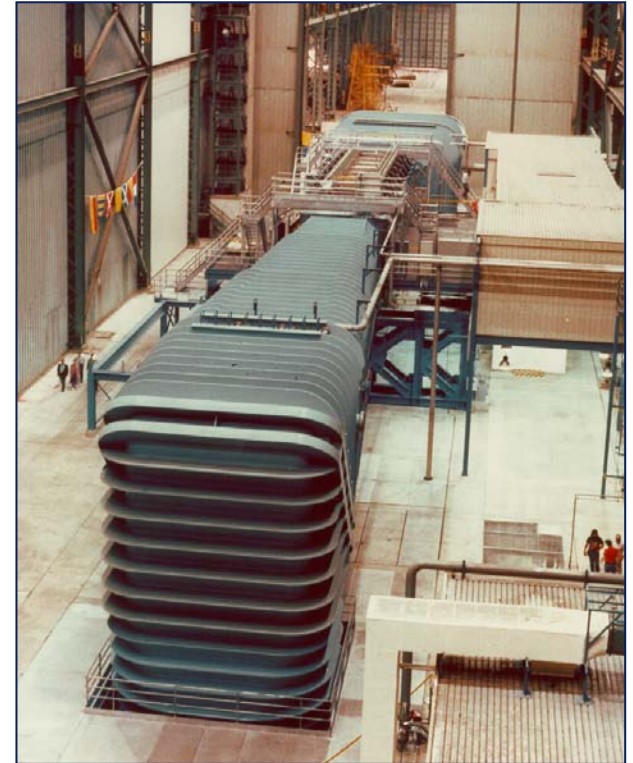
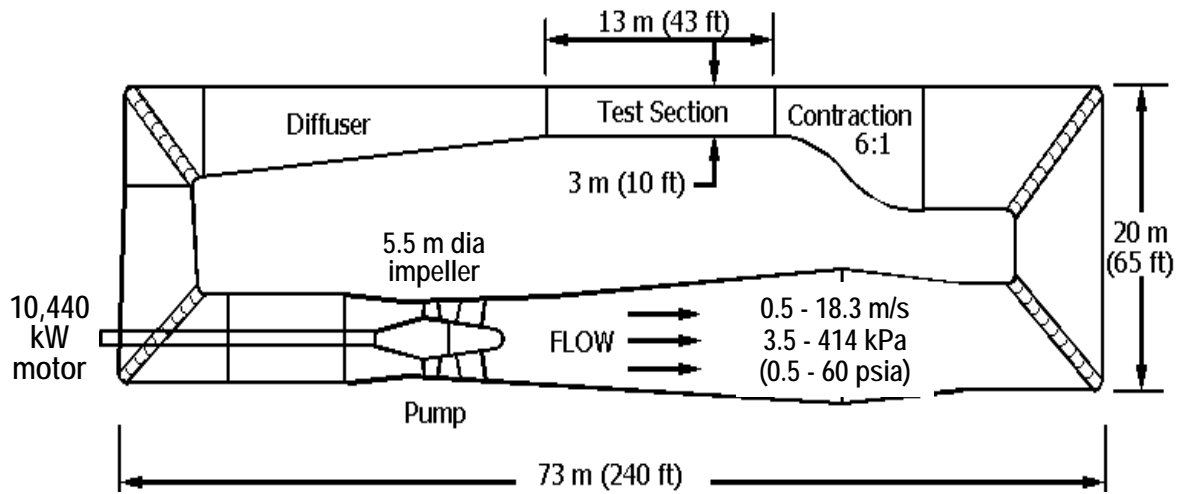
An oil pipeline



Noise sources = random laser excitations

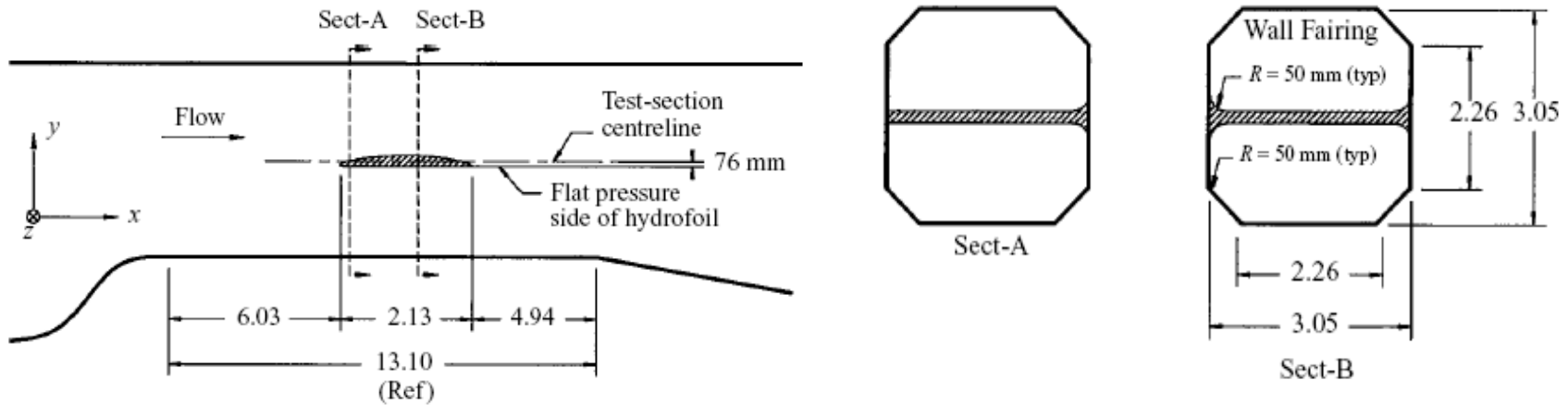
965mm-long, empty steel pipe. Outer diameter=48 mm. Thickness=4.5 mm. [10-30kHz]

# Experimental Set-up: Test Facility

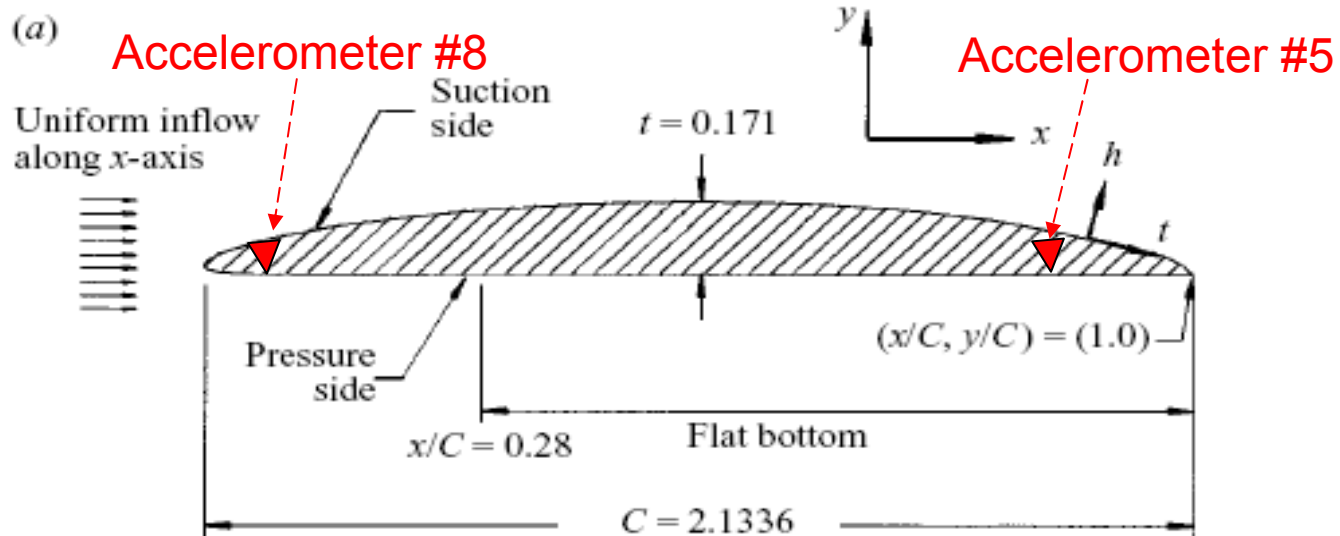


U.S. Navy's William B. Morgan Large Cavitation Channel,  
Memphis, TN

# Test section & Hydrofoil Profile



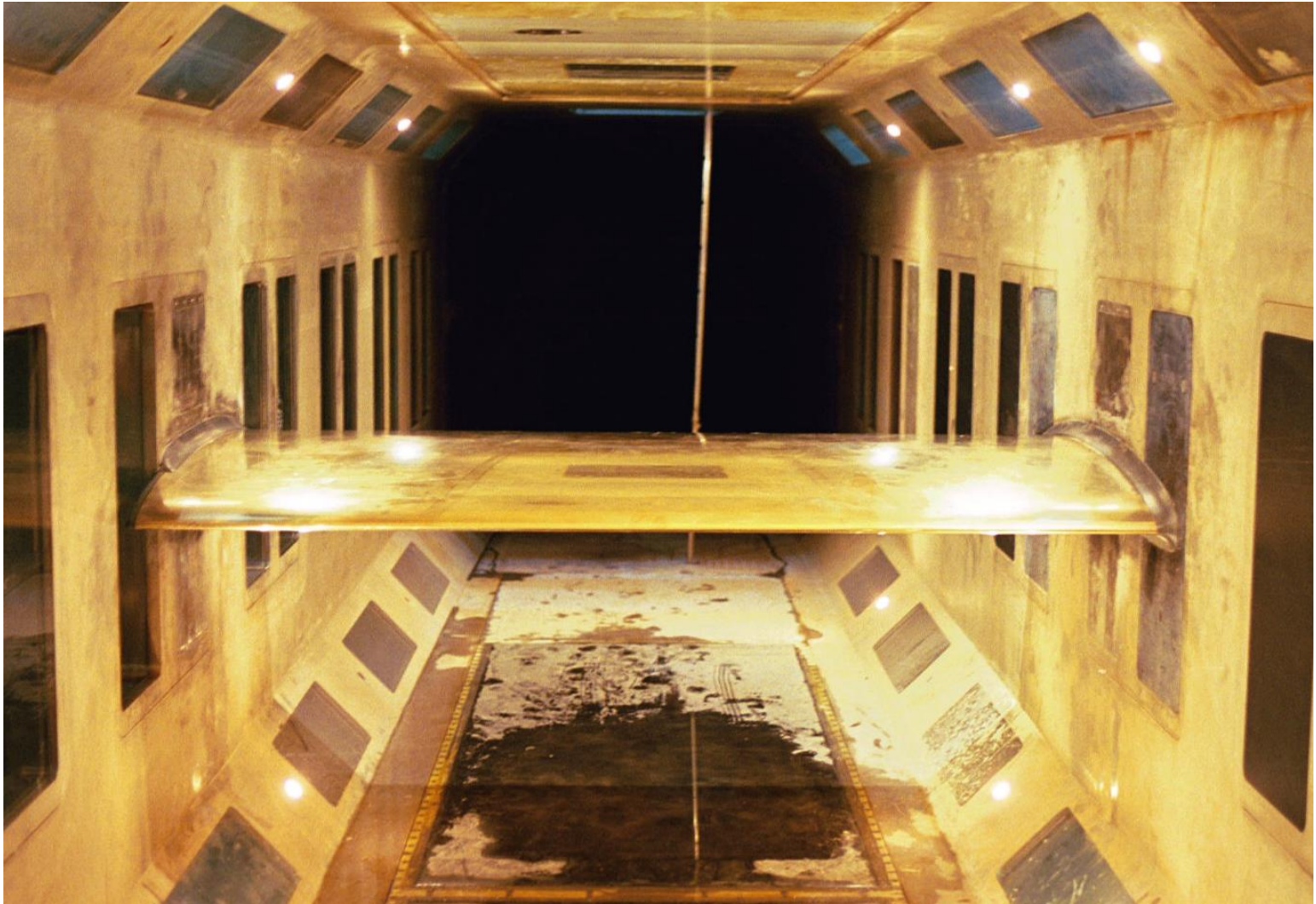
Flow Speed 18.3m/s. Chord-based Reynolds Number  $\sim 50$  Million



Figures reproduced from: Bourgoyne, et al. *JFM*, 2003.



# Hydrofoil installed in the LCC

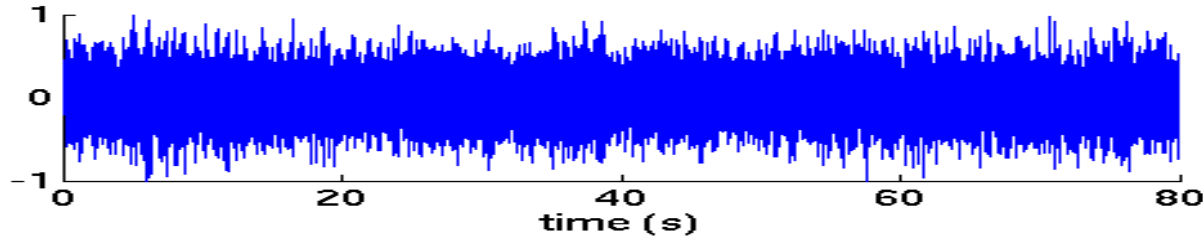


(view looking downstream)

# Spectrum of random vibrations generated by the turbulent boundary layer

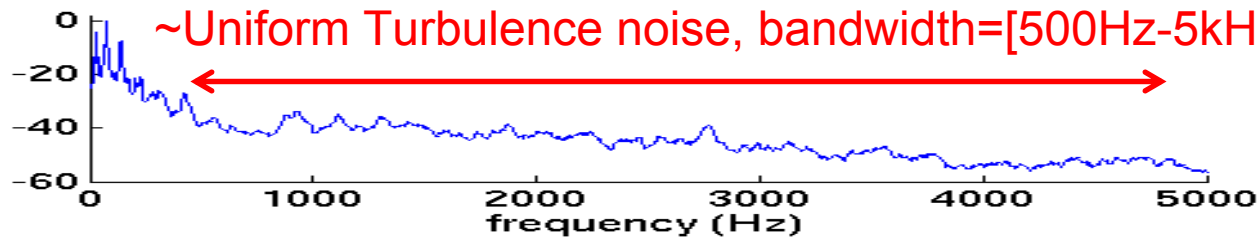
*Sensor Separation,  $D=1.77m$ . Each Noise Recording duration=1 min.*

RUN: AC0268.txt. Accelerometer: #8

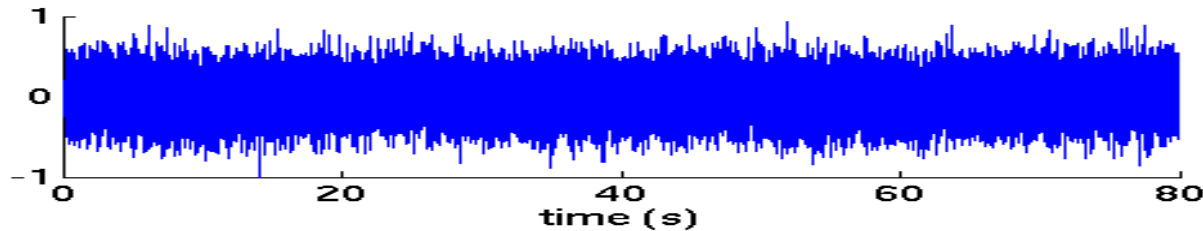


Accelerometer # 8

Leading Edge

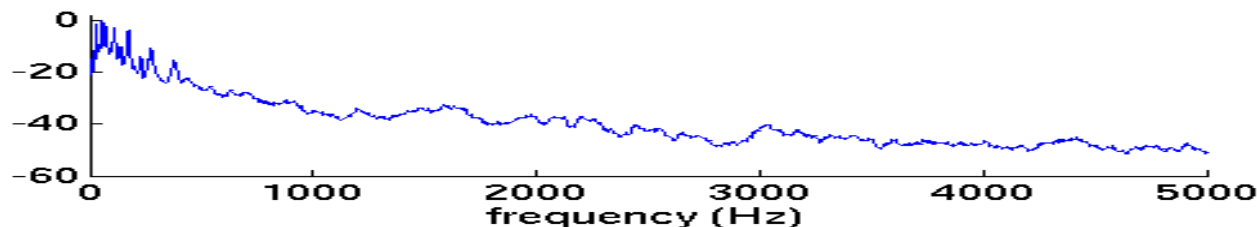


RUN: AC0268.txt. Accelerometer: #5

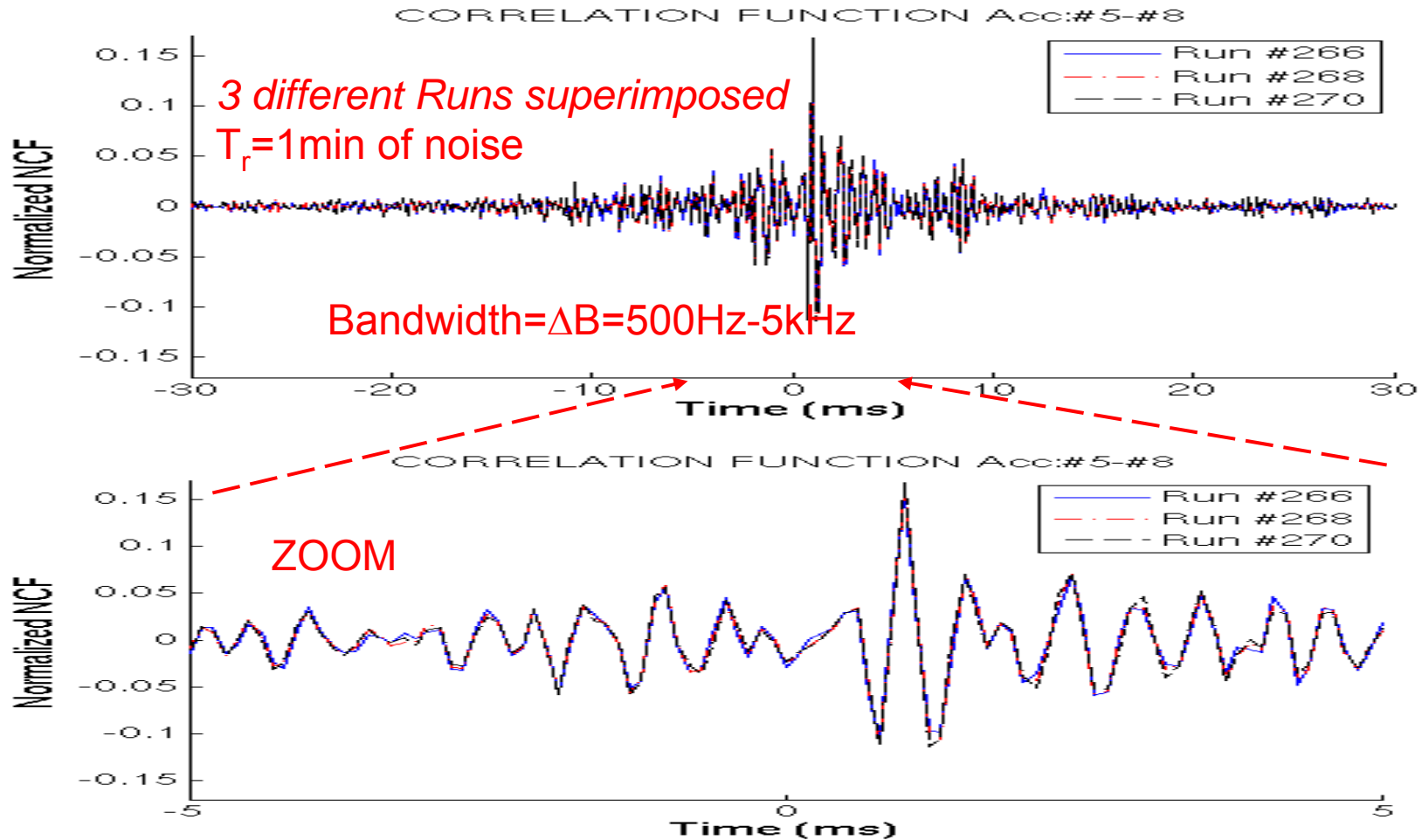


Accelerometer # 5

Trailing Edge



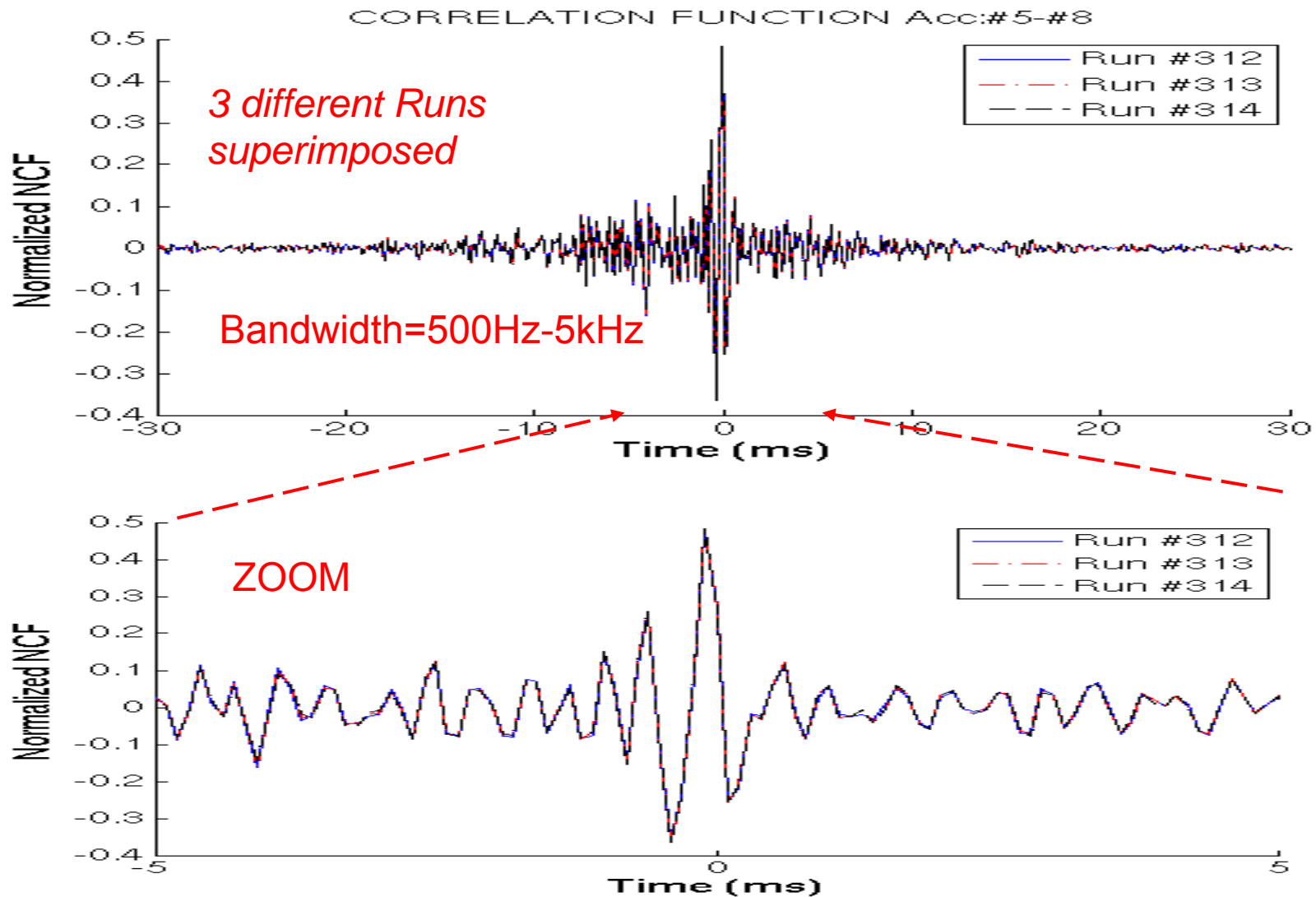
# Cross-Correlation Function: Normal Mounting



- Coherent and identical time-signatures emerge from the noise cross-correlation function between the accelerometer pair using three different recordings,  $T_r=1$  min. This time signature corresponds to the structural Green's function between the two accelerometer locations.
- Emergence rate of the coherent signature:  **$\text{SNR} \equiv \text{sqrt}(T_r \Delta B)$**

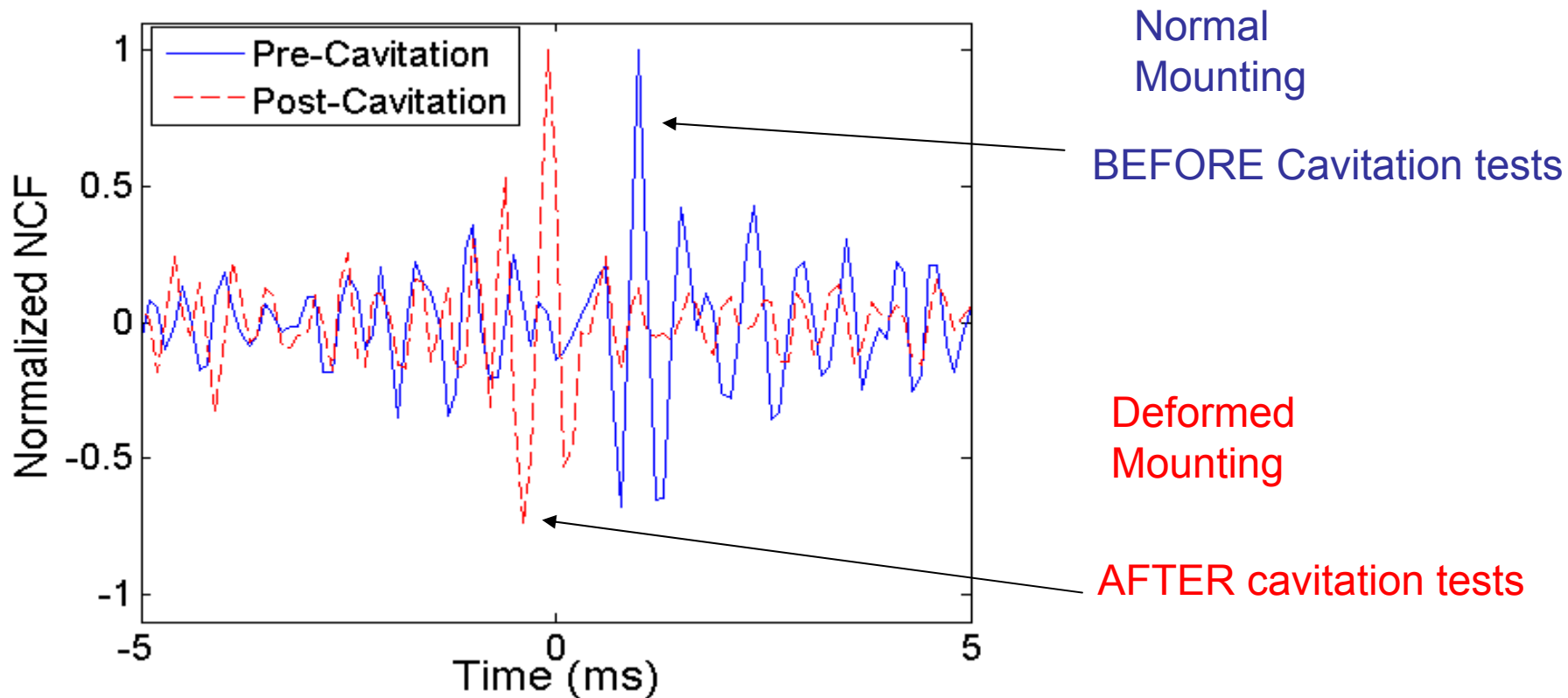


# Cross-Correlation Function: Deformed Mounting



Again a consistent temporal signature of the noise cross-correlation function emerges, but it differs from the pre-cavitation test signature.

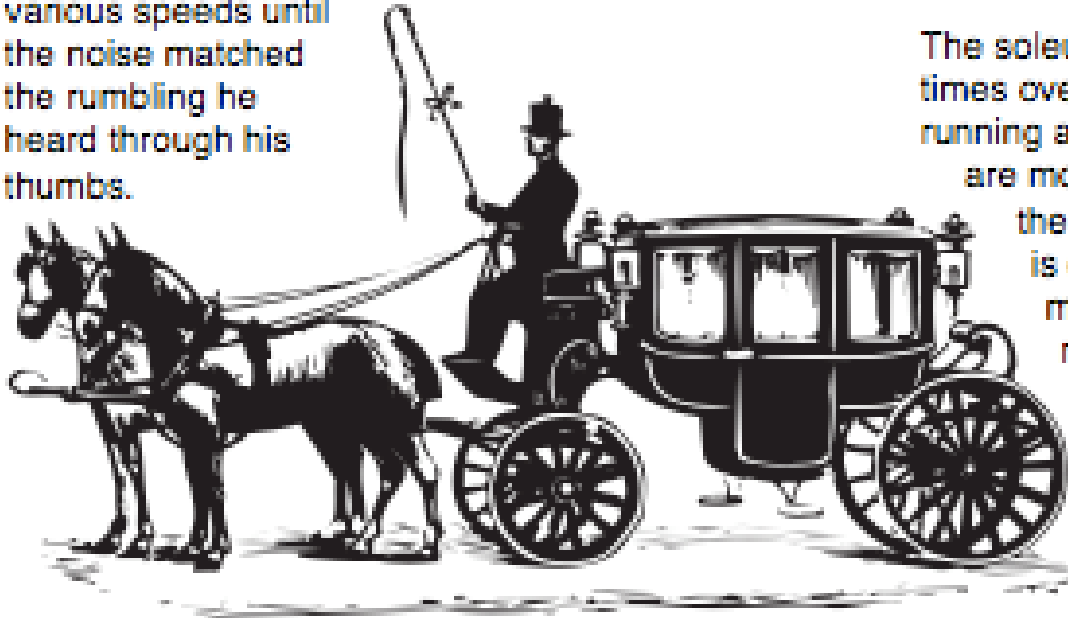
# Monitoring Structural Changes



Variations in the temporal structure of the noise cross-correlation function reveals that structural changes in the hydrofoil and its mounting have occurred because of the short-but-intense cavitation tests (Deformed Mounting was nearly equidistant from the two accelerometers).

# Muscle Noise

In 1810, the British physicist, physician and chemist, William Hyde Wollaston, compared the muscle sounds to the distant rumble of carriages over the cobblestone streets of London. To check his work, he had his carriage driven through the streets at various speeds until the noise matched the rumbling he heard through his thumbs.



Knowing the size of the cobblestones, and the wheel diameter, he was able to calculate the muscle sounds to be about 25 Hertz.

This fact in sounds can muscle is the least si angle betw 115 degree

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## *Partial History of the Fourier Transform*

**Horse-Spectrometer (1810)**

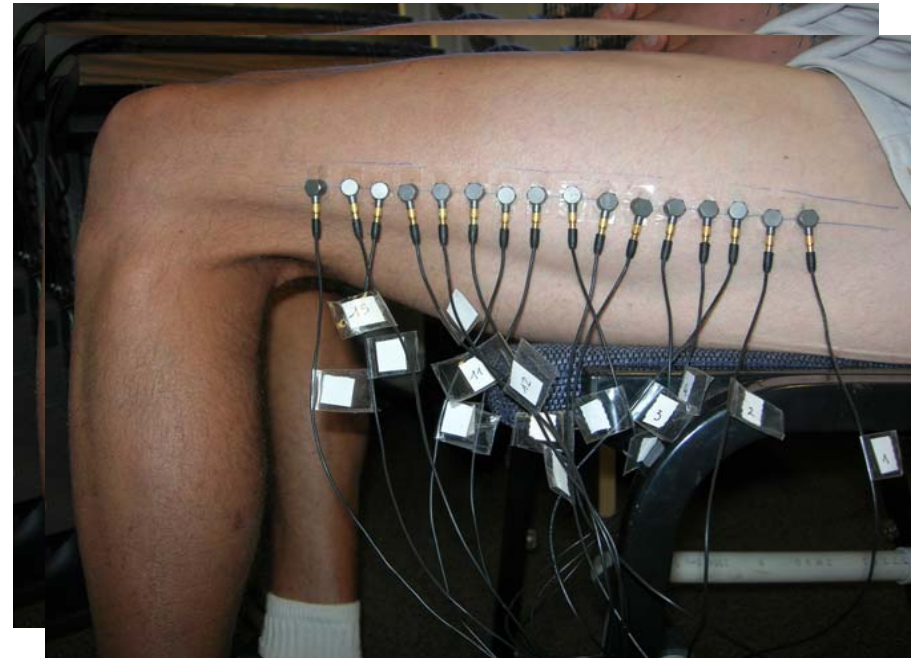
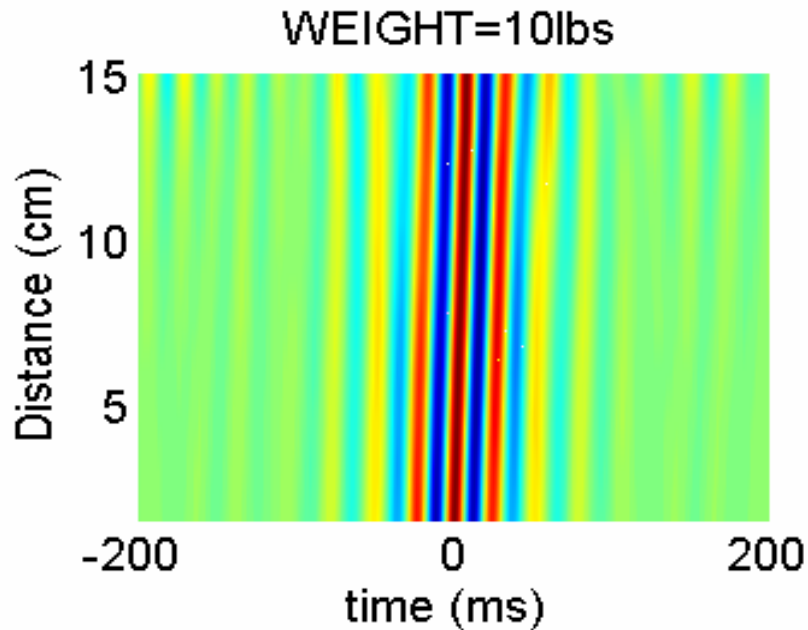
**Fourier (1810): published in 1822**

# Passive in-vivo Elastography from skeletal muscle noise

Karim G. Sabra, Stephane Conti,  
Philippe Roux, William A. Kuperman

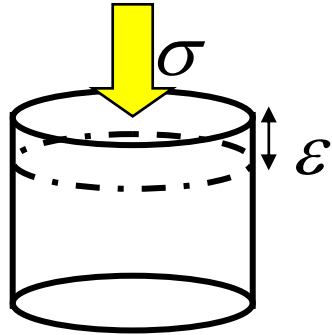


*Marine Physical Laboratory, Scripps, UCSD*



# Elastic Properties of Tissues

$$E = \frac{\sigma}{\varepsilon}$$



$$E = \mu \frac{3\lambda + 2\mu}{\lambda + \mu} \approx 3\mu$$

$\lambda$  et  $\mu$  Lamé coefficients

Biological Tissues :  $\lambda = 2,5 \text{ GPa}$ ,  $\mu = 25 \text{ kPa}$

Human Body

H.F. (1-50 MHz)

Fluid (Compressionnal waves)

- sound speed slight changes  $\sim 1500 \text{ m.s}^{-1}$

- Echogenicity imaging  $Z = \rho c$

$$c = \sqrt{\frac{\lambda + 2\mu}{\rho}} \approx \sqrt{\frac{\lambda}{\rho}}$$

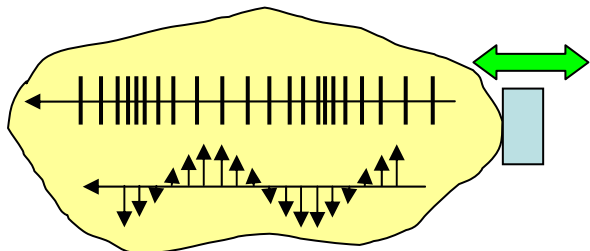
B.F. (1-100Hz)

Elastic Solid (Compression + Shear waves)

- Shear wave speed is slow  $\sim 1-5 \text{ m.s}^{-1}$

- centimetric wavelengths

$$v = \sqrt{\frac{\mu}{\rho}}$$



# MechanoMyoGrams (MMG)

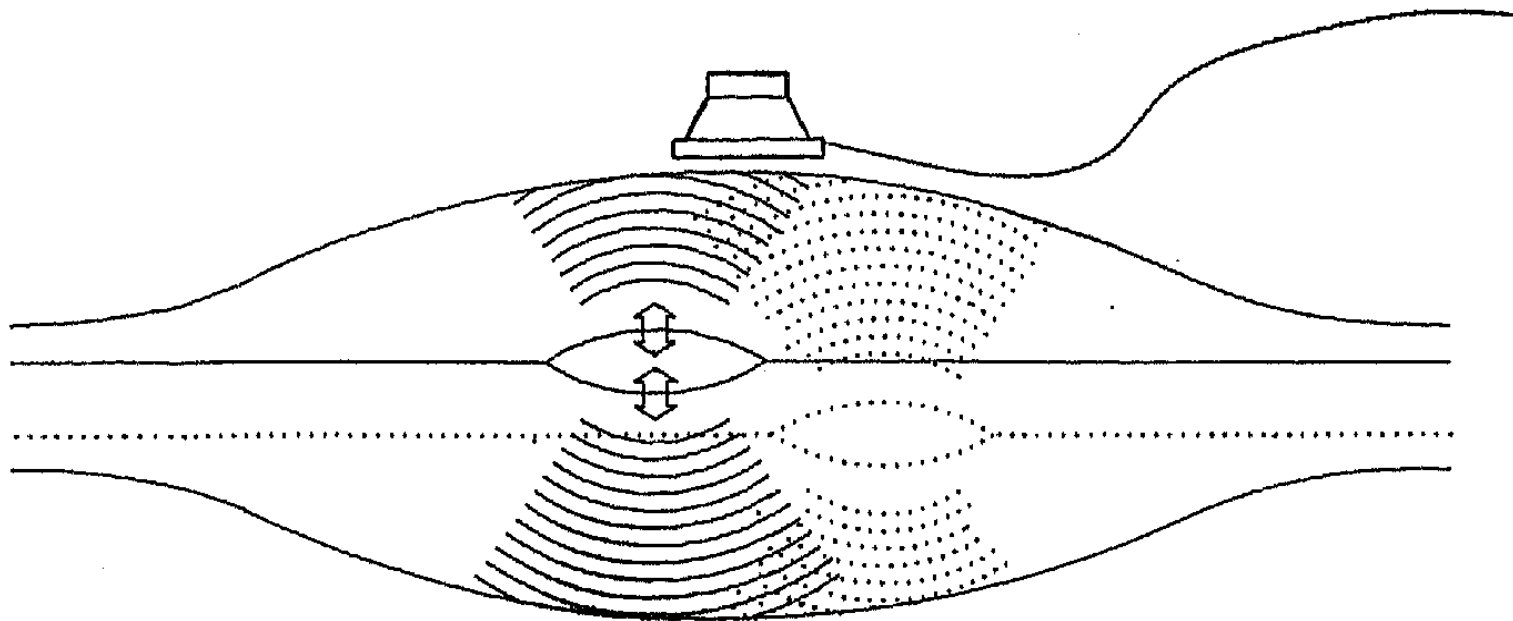


Figure 11.3. Schematic representation of the hypothesized MMG generation process.

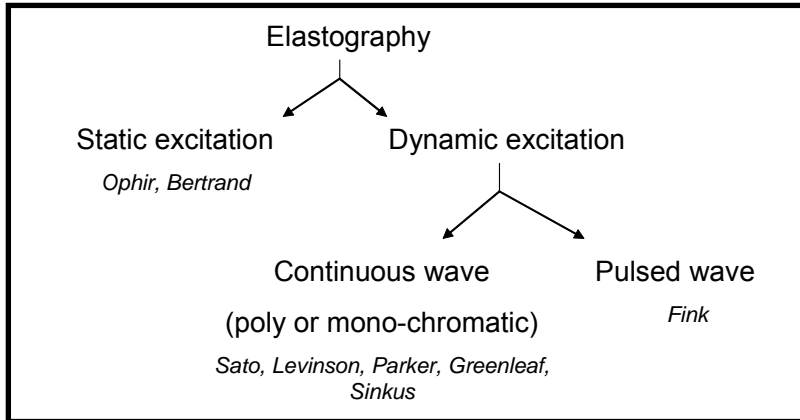
MMG result from vibrations generated by dimensional changes of the active muscle fibers during (fluctuations of ) voluntary contractions (*Orizzio 93*)

—————→ *Are shear waves generated naturally ?*

# Motivation

## Shear wave excitation techniques used in active elastography

### ACTIVE

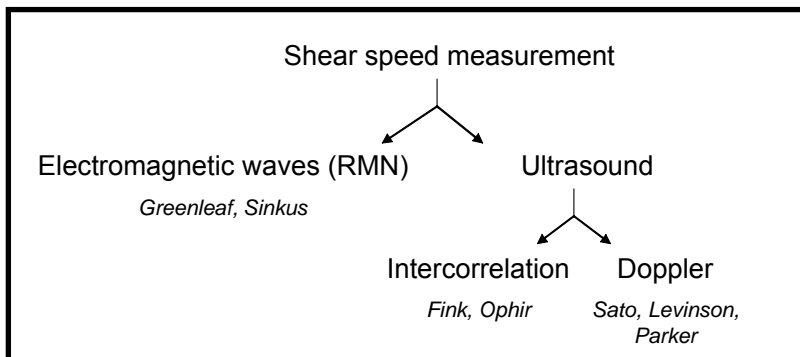


### PASSIVE

Use random-vibrations generated by the human body itself (e.g. muscle twitches)

## Shear speed measurement techniques

### ACTIVE



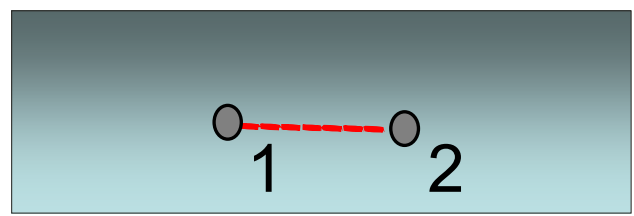
### PASSIVE

“Surface mechanomyograms” using skin-mounted accelerometers

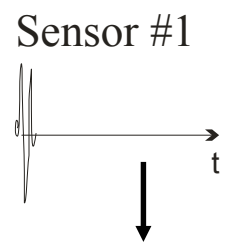


# Extracting Green's Function from Diffuse Wavefields

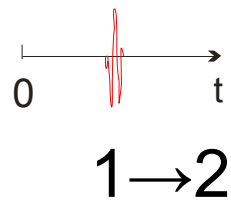
## ACTIVE



Sensor 1 broadcasts. Sensor 2 records.

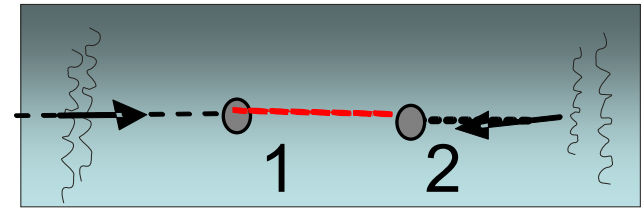


*Active transmission*

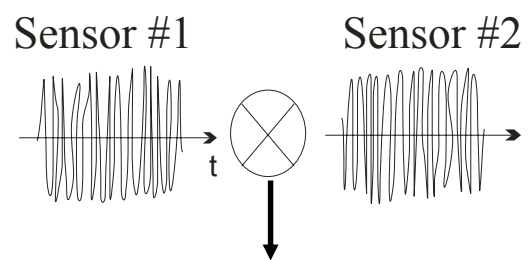


Green's Function ( - - - )

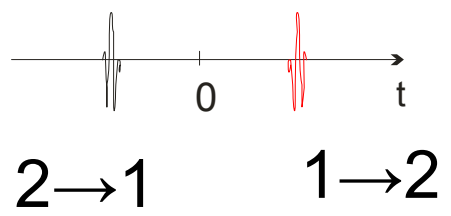
## PASSIVE



Diffuse field recorded at sensors 1 & 2.



*Cross-correlation between 1 and 2*



Green's Function Estimate ( - - - )

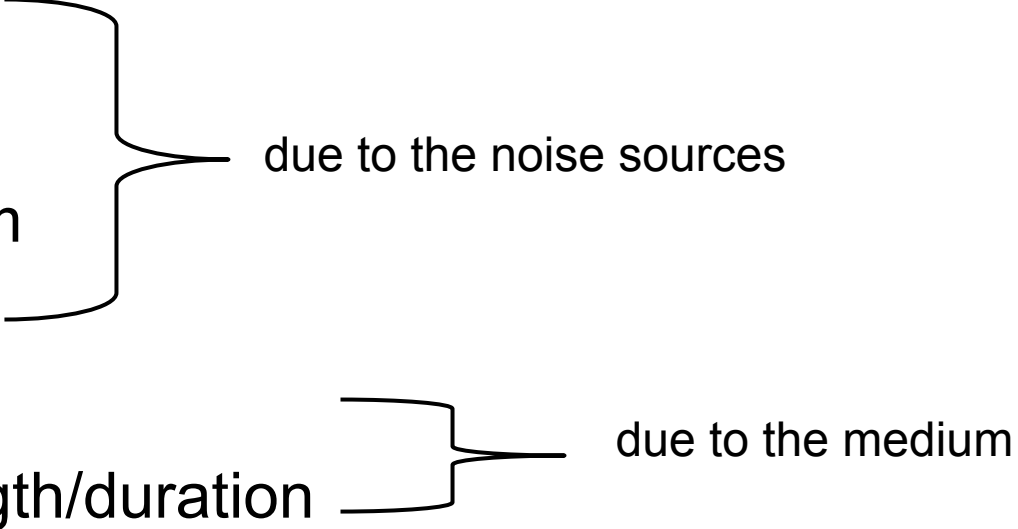
$$C_{12}(\tau) = \int_{-\infty}^{\infty} P(\mathbf{r}_1, t) P(\mathbf{r}_2, t + \tau) dt.$$

# Theoretical & Practical Issues

*Formal relationship for diffuse fields cross-correlations:*

$$C_{12}(\omega) = \beta \text{Imag}(G_{12}(\omega)) = \beta (G_{12}(\omega) - G_{12}^*(\omega)) / 2i$$

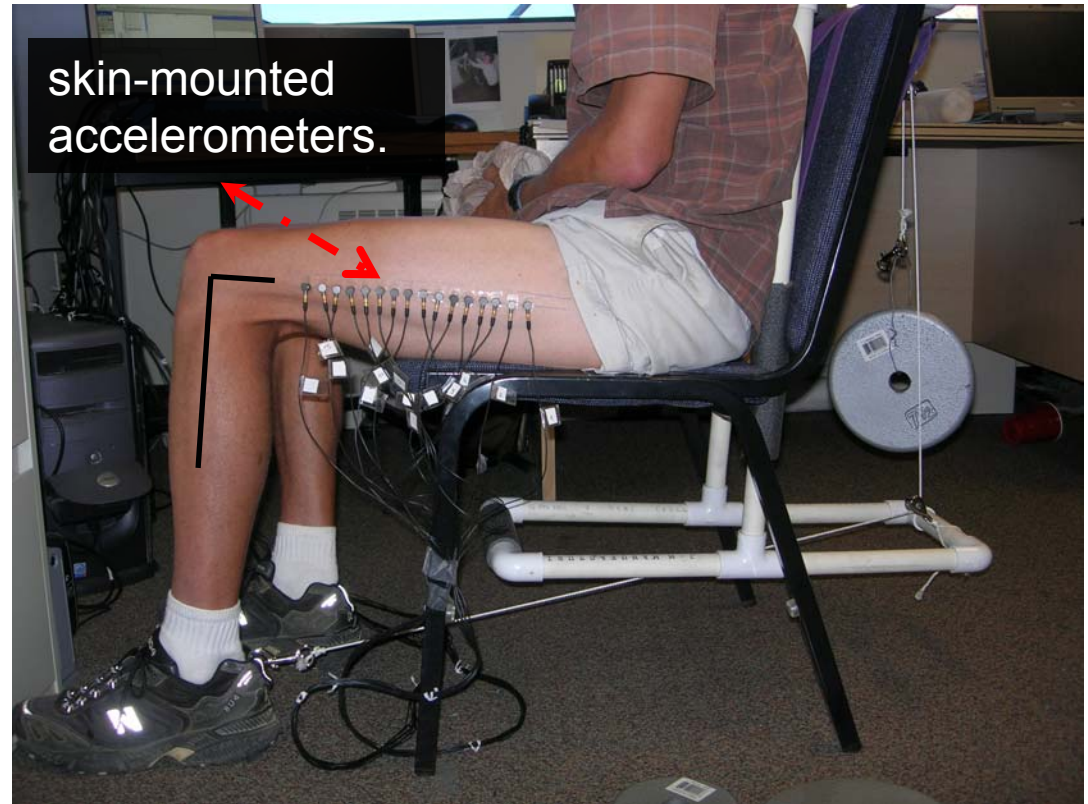
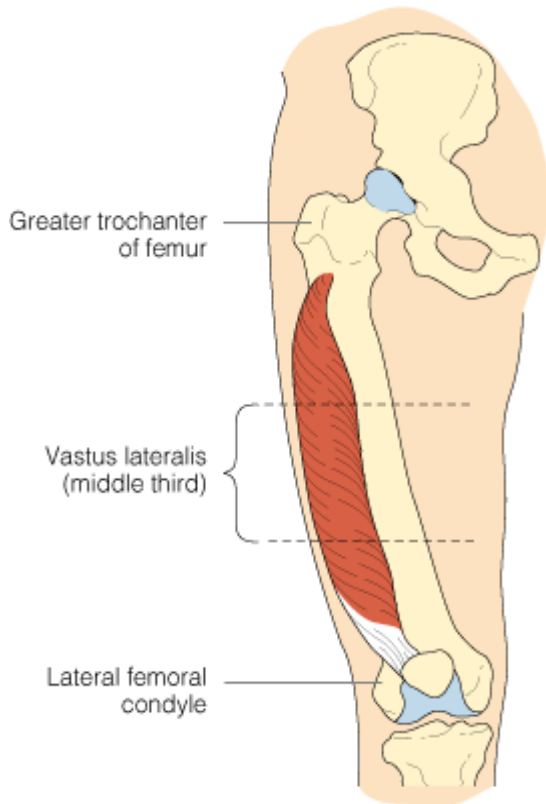
Limiting Factors:

1. Distribution
  2. Directionality
  3. Power spectrum
  4. Statistics
  5. Attenuation
  6. Coherence length/duration
- 

$$T_{\text{dispersion}} < T_{\text{statistical}} < T_{\text{recording}} < T_{\text{fluctuation}}$$

On short time-scale ( $< T_{\text{fluctuation}}$ ), the cross-correlation process is stationary

# Experimental Set-up

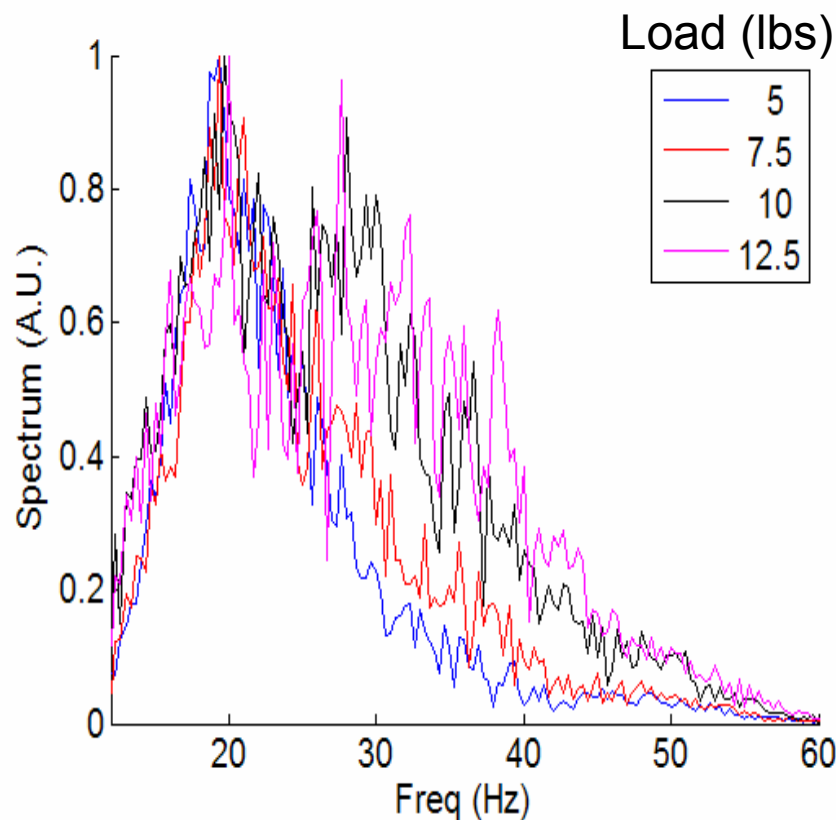
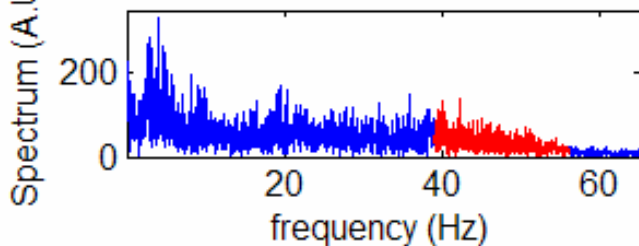
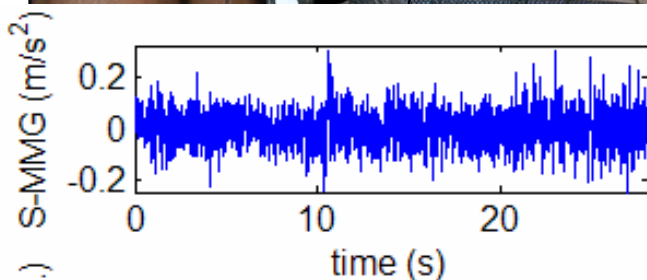
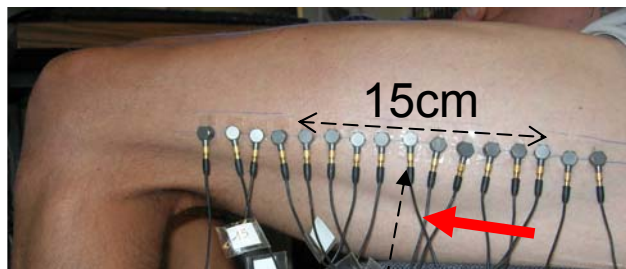


Miniature (1.5 gm), ceramic shear ICP® accel., 100 mV/g,

- Method: Isometric contractions of the vastus lateralis (knee extensor ) muscle
- Goal: Relate “muscle hardness” (shear modulus) to weight load (produced effort/torque)

# Surface Mechanomyograms (MMG)

Lifted weight= 10lbs

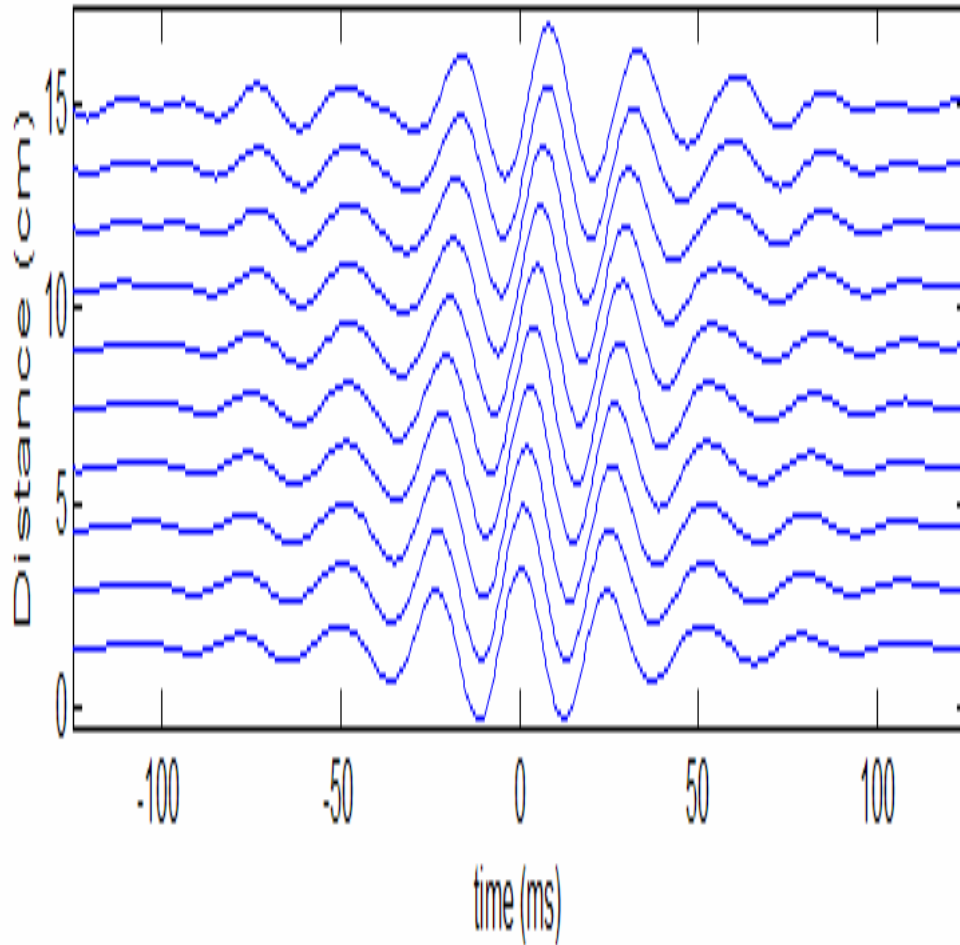


Spectral shift towards higher frequencies & increase of “rms” value with increasing effort due to:

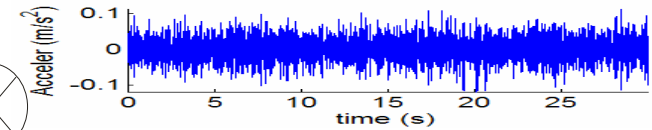
1. Recruitment of faster motor units
2. Increase in firing rates of motor units

(Shinoara,98)

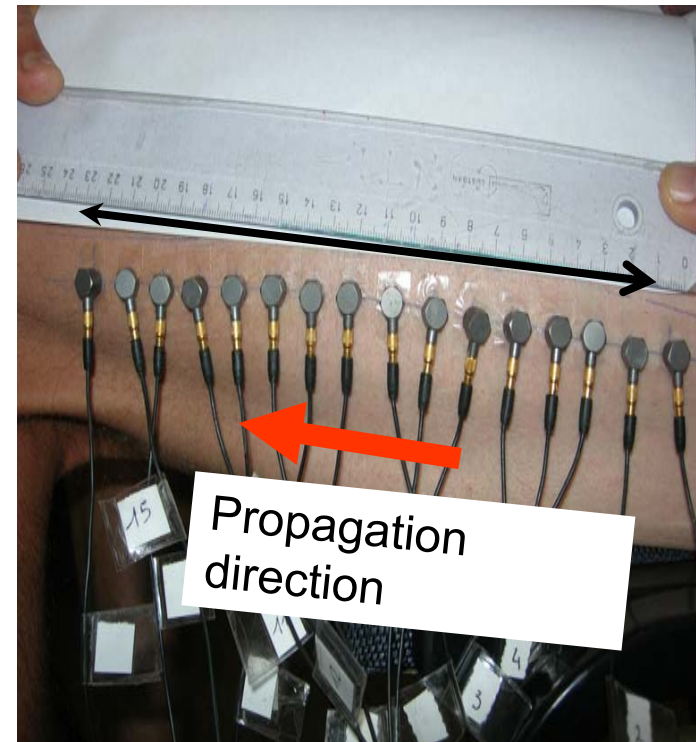
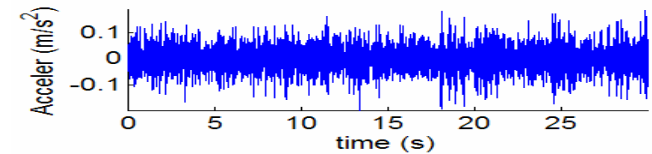
# Emergence of Coherent Shear Waves



Sensor #1.



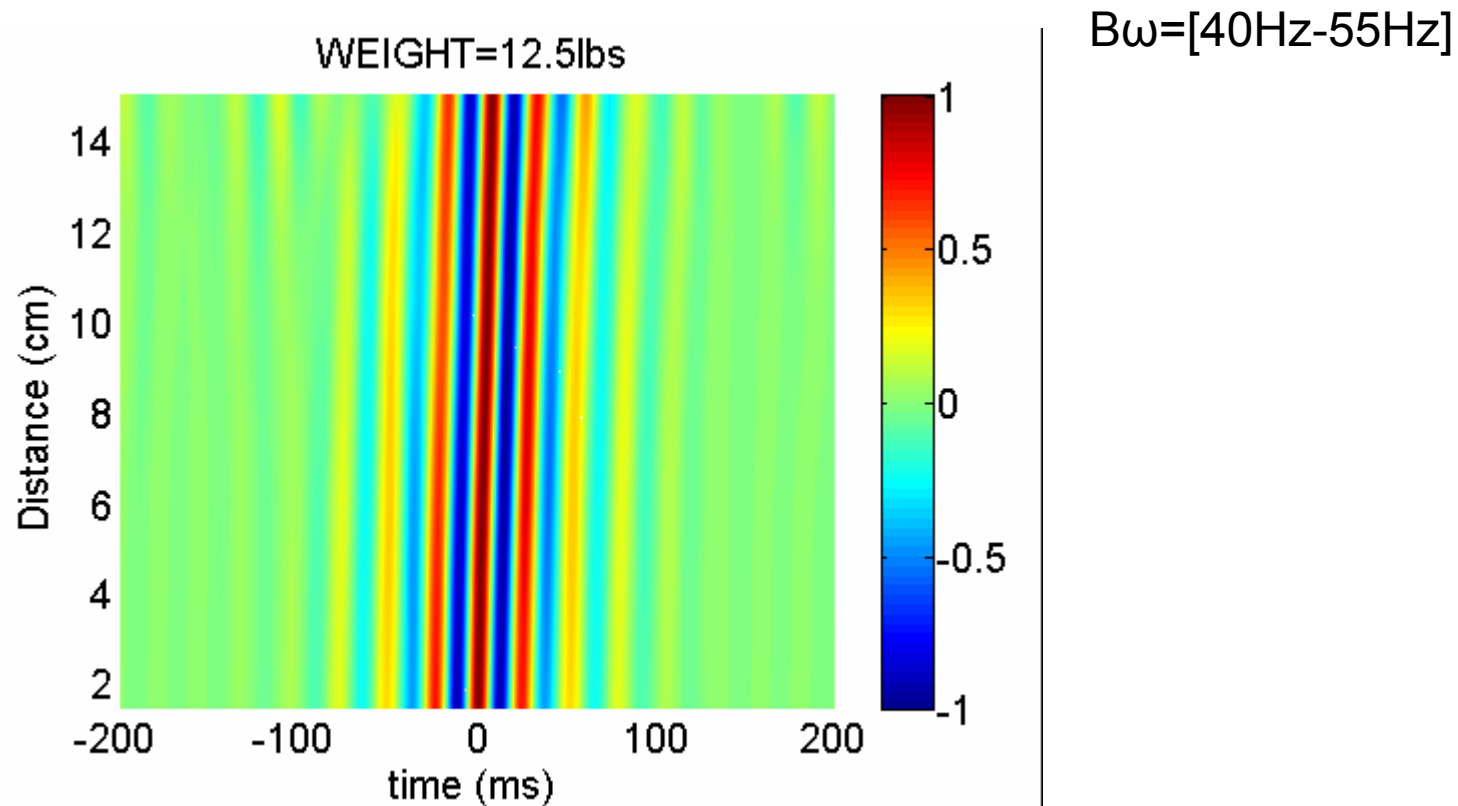
Sensor #11.



Weight=10lbs.  $B\omega=[40\text{Hz}-55\text{Hz}]$ .  
30sec of muscle noise

# Coherent Shear Wave Profile

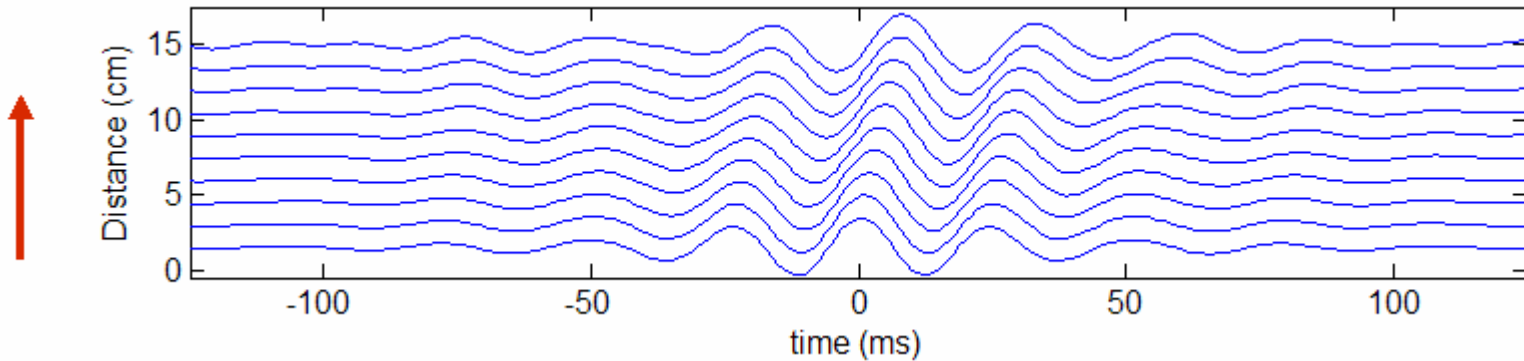
Average profile over all equidistant pairs



- Only use sensors mounted on the middle third of the vastus lateralis muscle
- Pennation angle  $\sim 5$  degrees. [Winter, 1990]



# Shear Wave Speed Dispersion

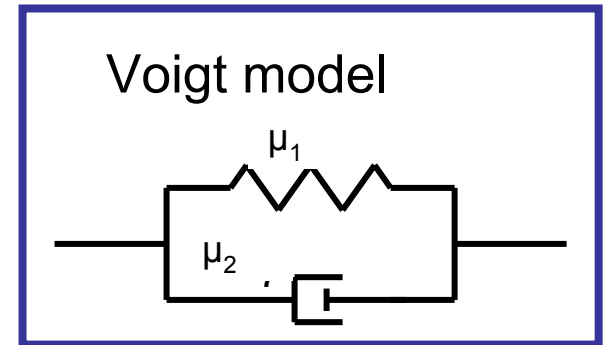


## Voigt model

$$-\rho\omega^2 S = (\mu_1 + i\omega\mu_2)\Delta S$$

$\mu_1$ : Shear modulus (kPa)

$\mu_2$ : Shear viscosity (Pa.s)



$$k = \sqrt{\frac{\rho\omega^2}{\mu_1 + i\omega\mu_2}}$$

$$c_s = \sqrt{\frac{2(\mu_1^2 + \omega^2\mu_2^2)}{\rho(\mu_1 + \sqrt{\mu_1^2 + \omega^2\mu_2^2})}}$$

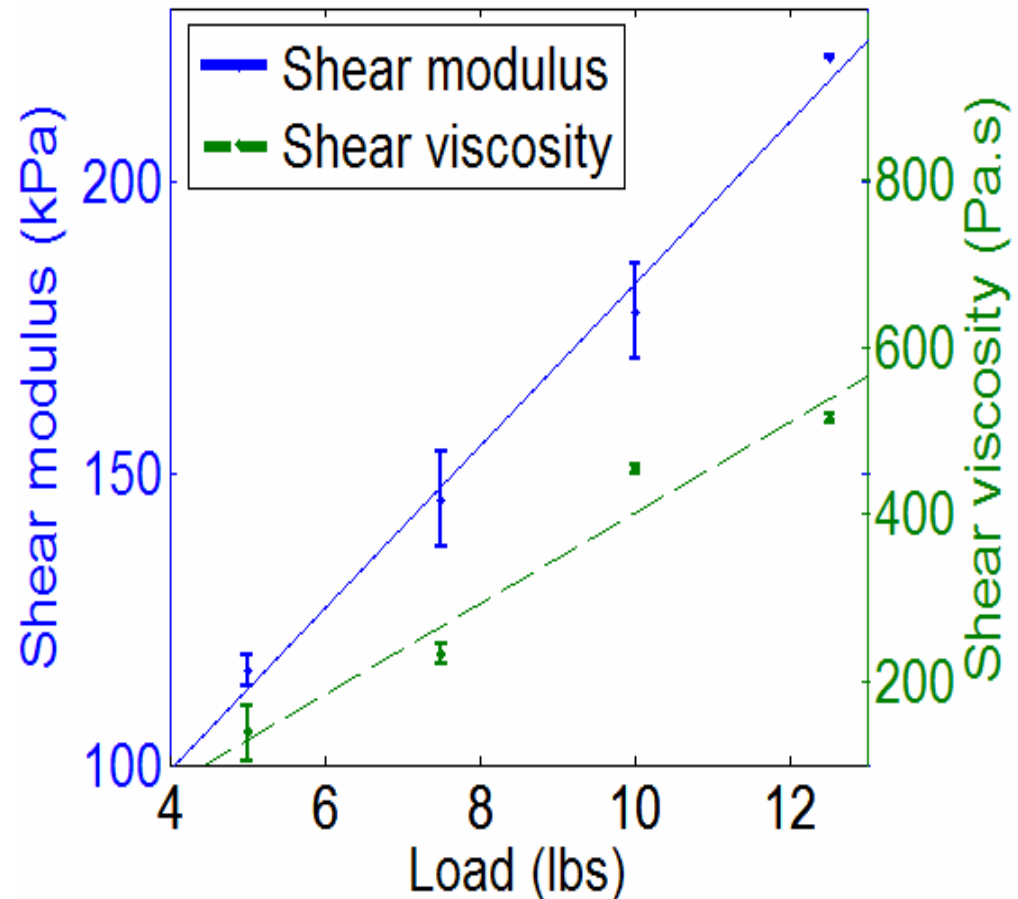
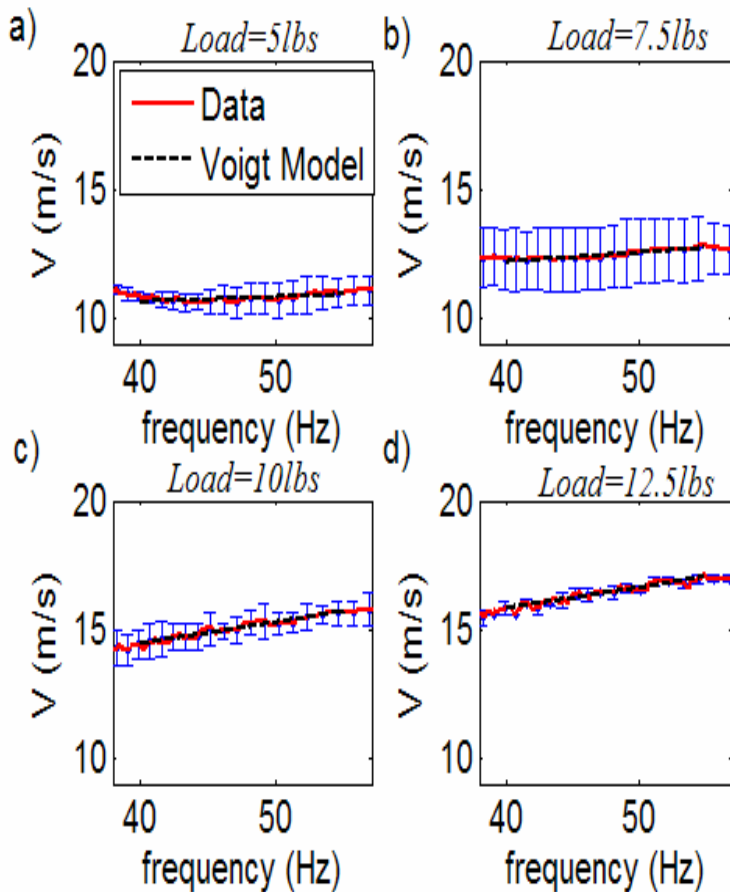
For isotropic elastic (small displacement), locally homogeneous tissue

# Viscoelastic parameters vs. load

**Voigt model**

$$c_s = \sqrt{\frac{2(\mu_1^2 + \omega^2 \mu_2^2)}{\rho(\mu_1 + \sqrt{\mu_1^2 + \omega^2 \mu_2^2})}}$$

$\mu_1$  : Shear modulus (kPa),  
 $\mu_2$  : Shear viscosity (Pa.s),



# Conclusions

- AMBIENT NOISE IS NOT ALWAYS A NUISANCE
- SOMETIMES THE NOISE IS THE SIGNAL
- COHERENT STRUCTURES CAN BE BUILT UP FROM NOISE CORRELATION
  - The time-bandwidth product of the recordings governs the accuracy of the results:  $SNR \equiv \sqrt{T_r \Delta B}$ .

## NOISE CAN BE

- USED FOR INVERSION
- USED FOR NON DESTRUCTIVE TESTING
- USED FOR IN SITU MONITORING OF STRUCTURES
- USED FOR PASSIVE MONITORING OF HUMAN BODY
- CAN BE ADDED FOR DETECTION OF WEAK SIGNAL