Radar/Sonar

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- Radar (RAdio Detection And Ranging) / Sonar (SOund NAvigation and Ranging -active)
- Such systems have been applied to a wide variety of applications
  - Military
    - Detection
    - Estimation
      - Localization, classification
    - Tracking
  - Commercial
    - fish location
    - bottom mapping (bathymetry)
    - navigation aids
    - seismic prospecting
- Each relies on the processing of reflected (electromagnetic/acoustic) energy
  - conintuous wave (CW) sinusoids
  - pulsed waveforms
- Reflections desired and undesired
  - desired reflections off of "targets of interest"
    - interest determined by application
    - what's noise to one is signal to another
  - undesired reflections
    - clutter / reverberation
    - together with system noise make up what's called ambient noise
- Extraction of desired information from NOISY measurements
  - the noise is what keeps us employed
  - without it there would be little need for signal processing
- Various types of Radar/Sonar systems
  - Monostatic
    - transmitter and receiver in same location
  - Bistatic
    - transmitter and receiver in different locations
  - -Multistatic
    - multiple transmitters (typically at different carrier frequencies) and multiple receivers all in possibly different locations, but observing the same space
    - sensor fusion - process which attempts to combine information from multiple receivers to extract desired information about observation space

Narrowband Signals and Characterization of Bandlimited Systems

- When the echo energy arrives at the receiver the narrowband (NB) waveform is filtered through some transduction device (antenna, hydrophone, microphone, etc.).
- A signal is NB when $X(f) = 0$ for $|f \pm f_0| > W$, $f_0 \gg W$.
- Frequency response of the receiver $H(f) = |H(f)|e^{j\angle H(f)}$
- Ideally $H(f) = Ke^{-j2\pi f t_0}$ (distortionless)
  - input $\tilde{s}(t)$ yields output $K \cdot \tilde{s}(t - t_0)$
- Define the bulk time shift as

$$\boxed{\tau(f) = -\frac{1}{2\pi}\frac{d\angle H(f)}{df}} \tag{1}$$

- Distortionless $H(f) \to |H(f)| = K_1$ constant and $\tau(f) = K_2$ constant
- $|H(f)| \neq K_1 \to$ Amplitude distortions
  - intentional such as in bandpass filtering
    - ok if signal energy in passband
  - non-intentional amplify/attenuate portion of signal spectrum
  - effects in time domain anticipatory
- $\tau(f) \neq K_2 \to$ Phase distortions
  - effects in time domain non-anticipatory
  - can at times severely distort waveform
    - Equalizer $1/H(f)$ used in Digital communication systems
      - restores undistorted signal and prevents intersymbol interference (ISI)
  - If we can approximate $H(f)$ in support band of waveform, then we can at times correct distortions if problematic
    - don't care about $H(f)$ outside of signal support band
- Assume we can approximate $H(f)$ in the vicinity of carrier as

$$H(f) = |H(f_0)|e^{j\angle H(f)} \tag{2}$$

where phase is approximated via a Taylor series about the carrier

$$\angle H(f) = \angle H(f_0) + \left.\frac{d\angle H(f)}{df}\right|_{f=f_0}(f - f_0) + \frac{1}{2}\left.\frac{d^2\angle H(f)}{df^2}\right|_{f=f_0}(f - f_0)^2 + \text{h.o.t} \tag{3}$$

- $\angle H(f_0)$ simply shifts phase of input signal

  - not a time delay in general
  - phase shifts can appear as time delays for pure sinusoidal inputs

$$|H(f_0)| \cdot cos\,(2\pi f_0 t + \angle H(f_0)) = |H(f_0)| \cdot cos\,[2\pi f_0(t - T_p(f_0))] \tag{4}$$

    where $T_p(f_0) \triangleq -\angle H(f_0)/(2\pi f_0)$

- The bulk time delay is given by

$$\begin{aligned} \tau(f) = -\frac{1}{2\pi}\frac{d\angle H(f)}{df} &= \left[-\frac{1}{2\pi}\left.\frac{d\angle H(f)}{df}\right|_{f=f_0}\right] + \left[-\frac{1}{2\pi}\left.\frac{d^2\angle H(f)}{df^2}\right|_{f=f_0}(f - f_0)\right] \\ &\triangleq T_g(f_0) + \beta(f)(f - f_0) \\ &\triangleq T_g(f_0) + T_{disp}(f) \end{aligned} \tag{5}$$

- Bulk time delay consist of contributions from two terms:
  - group (or envelope) delay $T_g(f_0)$
    * delays entire received input signal
  - dispersion $T_{disp}(f)$
    * Frequency dependent time delay
      · $\beta(f) > 0 \rightarrow \angle H(f)$ concave down $\rightarrow$ higher frequency components get delayed most, arrive later
      · $\beta(f) < 0 \rightarrow \angle H(f)$ concave up $\rightarrow$ higher frequency components get delayed least, arrive sooner

- If no dispersion and $\tilde{x}_0(t)$ is complex envelope (CE) of input to receiver with frequency response $H(f)$, then
  - CE of demodulated output given by
    * $\tilde{y}_0(t) = |H(f_0)|e^{j\angle H(f_0)}\tilde{x}_0[t - T_g(f_0)]$
  - recall that temporal effects of dispersion are rarely intuitive
    * can be intuitive in the context of pulse compression of LFM signals
    * more later

- We will arbitrarily assume that dispersion term is negligible
  when its maximum contribution to the phase shift of the output spectrum is less than $\pi/4$:

$$2\pi\left(\frac{\beta(f_0)}{2}\right)W^2 \leq \frac{\pi}{4} \tag{6}$$

- This is equivalent to requiring its maximal contribution to bulk time delay to be less than $1/(4W)$
  - somewhat conservative given that the signal cannot change any faster than approximately $1/W$

- 1/2 the Nyquist sampling period $1/2W$

- Significance of dispersion is determined by bandwidth of signal W

$$W \leq \frac{1}{2\sqrt{\beta(f_0)}} \tag{7}$$

- 2nd order system numerical example
  - good model for most transducers (High quality factor Q)

Radar/Sonar Signal Model

- Properties of reflected signal depend on target characteristics (shape, motion, location etc.)
  - typically experiences attentuation and a random phase shift
- Slowly fluctuating point target
  - "slowly fluctuating" $\rightarrow$ target characteristics fixed during illumination
    * effects of motion can modeled by a constant (fixed in time) Doppler shift during time of illumination
  - "point" $\rightarrow$ depth (in seconds) negligible relative to pulse length
    * can consider it a point reflector relative to pulse envelope
    * reflected signal is attenuated, delayed version of transmitted signal
- Hence, attenuation and phase shift constant during illumination
- Assume Radar/Sonar transmit a CW of average power P_t

$$s_t(t) = \sqrt{2P_t} cos(2\pi f_0 t) = \sqrt{2P_t} \cdot \mathrm{Re}[e^{j2\pi f_0 t}], \qquad -\infty < t < \infty \tag{8}$$

- Assume that at range $R = \tau c/2$ target consists of K reflecting surfaces
- Return signal given by

$$s_r(t) = \sqrt{2P_t} \cdot \mathrm{Re}\left[\sum_{i=1}^{K} g_i \cdot e^{j(2\pi f_0 (t-\tau)+\theta_i)}\right] \tag{9}$$

- g_i represent attenuation gains due to transmit/receive antenna gains, two-way path loss, Radar cross section of target
  - Radar/Sonar range equations predict such losses, see Skolnik, Urick
- θ_i random phases
- A powerful central limit theorem exists which says that if
  - θ_i's are statistically independent
  - g_i's are equal in magnitude
  - K is large (> 6 usually sufficient)

  then return echo is equivalently modeled as

$$s_r(t) = \sqrt{2P_t} \cdot \mathrm{Re}\left[\widetilde{b} \cdot e^{j2\pi f_0 (t-\tau)}\right] \tag{10}$$

  where the complex gain $\widetilde{b}$ can be chosen to model various target types
  - nonfluctuating target $\rightarrow \widetilde{b}$ constant attentuation gain
  - random fluctuating targets models
    * phase of $\widetilde{b}$ uniformly distributed

- - $|\tilde{b}|^2$ can assume various models depending on application/system
      - Swerling models (popular), Rician, Nakagami, etc.
  - Swerling II given by assumption that $\tilde{b} \sim CN(0, \sigma_{\tilde{b}}^2)$

- To extend our return echo model to arbitrary complex envelopes $\tilde{f}(t)$ we assume that
  - reflection process is essentially independent of frequency
    - not true in general, since model for $\tilde{b}$ depends on Radar target cross section
    - narrowband nature of signal saves us
  - reflection process is linear

- Assume that we transmit

$$s_t(t) = \sqrt{2E_t} \cdot \mathrm{Re}\left[\tilde{f}(t)e^{j2\pi f_0 t}\right] = \sqrt{2E_t} \cdot \mathrm{Re}\left[\int_{-\infty}^{\infty} \tilde{F}(f)e^{j2\pi(f_0+f)t}df\right] \tag{11}$$

$$\begin{aligned} s_r(t) &= \sqrt{2E_t} \cdot \mathrm{Re}\left[\tilde{b} \cdot \int_{-\infty}^{\infty} \tilde{F}(f)e^{j2\pi(f_0+f)(t-\tau)}df\right] \\ &= \sqrt{2E_t} \cdot \mathrm{Re}\left[\tilde{b} \cdot \tilde{f}(t-\tau)e^{j2\pi f_0(t-\tau)}\right] \end{aligned} \tag{12}$$

- Since we assume $\int_{-\infty}^{\infty} |\tilde{f}(t)|^2 dt = 1$, the received energy is given by
  - $E_r = E_t|\tilde{b}|^2$ for nonfluctuating targets and by $E_r = E_t\sigma_{\tilde{b}}^2$ for fluctuating targets
    - $\sigma_{\tilde{b}}^2 \triangleq E\{|\tilde{b}|^2\}$

- If target is moving then
  - Line of sight range is function of time $R(t)$
  - time delay for monostatic system given by $\tau(t) = 2R(t)/c$

$$\boxed{s_r(t) = \sqrt{2E_t} \cdot \mathrm{Re}\left[\tilde{b} \cdot \tilde{f}\left(t - \frac{2R(t)}{c}\right) e^{j2\pi f_0\left(t - \frac{2R(t)}{c}\right)}\right]} \tag{13}$$

- Assume linear model for range $R(t) = R(0) + R^{'}(0)t$ and consider carrier term

$$e^{j2\pi f_0\left(t - \frac{2R(t)}{c}\right)} \triangleq e^{j\phi(t)} \tag{14}$$

$$\begin{aligned} \phi(t) &= 2\pi f_0\left[t - \frac{2R(t)}{c}\right] = 2\pi f_0\left[t\left(1 - \frac{2R^{'}(0)}{c}\right) - \tau_0\right] \\ &= 2\pi\left(f_0 - \frac{2f_0R^{'}(0)}{c}\right)t - 2\pi f_0\tau_0 \\ &\triangleq 2\pi(f_0 + f_d)t - 2\pi f_0\tau_0 \end{aligned} \tag{15}$$

- Define the frequency shift in carrier as

$$f_d \stackrel{\triangle}{=\!=} -\frac{2f_0R^{'}(0)}{c} = -\frac{2R^{'}(0)}{\lambda_0} \tag{16}$$

$$s_r(t) = \sqrt{2E_t} \cdot \mathrm{Re}\left\{\tilde{b} \cdot \tilde{f}\left[t\left(1+\frac{f_d}{f_0}\right) - \tau_0\right] e^{j2\pi(f_0+f_d)t - j2\pi f_0\tau_0}\right\} \tag{17}$$

- Note three effects

  1. Compression ($f_d/f_0 > 0$) or stretching ($f_d/f_0 < 0$) of time axis
     - typically negligible in Radar, but can be significant in Sonar
     - signal does not change faster than $1/W$, thus negligible if

$$\left|\frac{2R^{'}(0)}{c}\right| \times \text{pulse width} \ll \frac{1}{W} \tag{18}$$

     - considering Fourier representation, one may considering constraining the phase contribution of f_d/f_0 to be less than $\pi/4$

$$\tilde{f}\left[t\left(1+\frac{f_d}{f_0}\right) - \tau_0\right] = \int_{-W}^{W} \tilde{F}(f) e^{j2\pi f\left[t\left(1+\frac{f_d}{f_0}\right)-\tau_0\right]} df \tag{19}$$

       this would require $|f_d/f_0| < 1/(2W \times \text{pulse width})$
       * more conservative
  2. A shift in carrier frequency called Doppler shift
     - used to estimate range rate (velocity) of target
     - useful classification parameter
  3. Time delay τ_0 in envelope and carrier
     - envelope delay used to estimate line of sight range from Radar/Sonar
     - carrier delay of use in coherent processing (e.g. SAR)
     - Both are very important for spatial processing

Matched Filters and Correlators

- Recall that received signal is given by

$$s_r(t) = \sqrt{2E_t} \cdot \mathrm{Re}\left\{\tilde{b} \cdot \tilde{f}(t-\tau_0)e^{j2\pi(f_0+f_d)t}\right\} \tag{20}$$

  here we assume time compression is negligible and absorb phase term into complex gain $\tilde{b}$

- Measurements are noisy
  - ambient noise
    - system (sensor) thermal noise
    - wind/waves, shipping
    - increasing transmit energy improves signal-to-noise ratio (SNR)
  - clutter / reverberation
    - unwanted reflections from other targets, sea surface/bottom
    - increasing transmit energy does not improve performance

- Measurement model given by

$$r(t) = \sqrt{2E_t} \cdot \mathrm{Re}\left\{\tilde{b} \cdot \tilde{f}(t-\tau_0)e^{j2\pi(f_0+f_d)t}\right\} + n(t) \tag{21}$$

  yielding complex envelope

$$\tilde{r}(t) = \sqrt{E_t} \cdot \tilde{b} \cdot \tilde{f}(t-\tau_0)e^{j2\pi f_d t} + \tilde{n}(t) \tag{22}$$

- How do we process $\tilde{r}(t)$ to extract desired information?
  - Optimal methods in Detection and Estimation theory exist
    - If noise is white Gaussian $\rightarrow$ optimal processor is linear (see Van Trees)

- Restrict attention to linear filters and design best $\tilde{h}(t)$
  - maximize output SNR

- Filter output given by

$$\tilde{y}(t) = \tilde{h}(t) * \tilde{r}(t) = \tilde{h}(t) * \tilde{s}_r(t) + \tilde{h}(t) * \tilde{n}(t) \stackrel{\triangle}{=} \tilde{y}_s(t) + \tilde{y}_n(t) \tag{23}$$

  and consists of (signal term) + (noise term). The output SNR is given by

$$SNR = \frac{|\tilde{y}_s(0)|^2}{|\tilde{y}_n(0)|^2} = \frac{\left|\int_{-\infty}^{\infty} \tilde{h}(a)\tilde{s}_r(-a)da\right|^2}{E\left\{\left|\int_{-\infty}^{\infty} h(-a)\tilde{n}(a)da\right|^2\right\}} = \frac{\left|\int_{-\infty}^{\infty} \tilde{h}(a)\tilde{s}_r(-a)da\right|^2}{N_0 \int_{-\infty}^{\infty} \left|\tilde{h}(a)\right|^2 \tilde{d}a} \tag{24}$$

$$\tag{25}$$

$$\leq \frac{\int_{-\infty}^{\infty} \left|\tilde{h}(a)\right|^2 da \int_{-\infty}^{\infty} |\tilde{s}_r(-a)|^2 da}{N_0 \int_{-\infty}^{\infty} \left|\tilde{h}(a)\right|^2 \tilde{d}a} = \frac{\int_{-\infty}^{\infty} |\tilde{s}_r(a)|^2 da}{N_0} = \frac{E_r}{N_0} \tag{26}$$

- Above inequality known as the Schwartz Inequality
- Equality obtained if and only if $\widetilde{h}(t) = \alpha \cdot \widetilde{s}_r^*(-t)$
  - known as Matched filter
    - filter matched in delay and Doppler
- Optimal signal detector is given by thresholding the statistic

$$L = |\widetilde{y}(0)|^2 = \left| \int_{-\infty}^{\infty} \widetilde{h}(-t)\widetilde{r}(t)dt \right|^2 = \left| \int_{-\infty}^{\infty} \widetilde{f}^*(t - \tau_0)e^{-j2\pi f_d t}\widetilde{r}(t)dt \right|^2 \tag{27}$$

- In general target's range/Doppler will be unknown a priori
- Must have a bank of filters matched to variety of target ranges and Doppler
  - See filter bank and range/Doppler gridded plane
  - Let true value of target time delay and Doppler be (τ_a, f_a)
  - Let our guess of target time delay and Doppler be (τ_0, f_d)
- Note that

$$\begin{aligned} \widetilde{y}(0) &= \widetilde{b}\sqrt{E_t} \int_{-\infty}^{\infty} \widetilde{f}(t - \tau_a)\widetilde{f}^*(t - \tau_0)e^{-j2\pi(f_d - f_a)}dt \\ &\triangleq \widetilde{b}\sqrt{E_t} \cdot \phi(\tau_0, f_d; \tau_a, f_a) \end{aligned} \tag{28}$$

- Define $\tau \triangleq \tau_0 - \tau_a$ and $f \triangleq f_d - f_a$
- Make the change of variables

$$t \to t - \frac{\tau_0 + \tau_a}{2} \tag{29}$$

- The following Radar/Sonar time-frequency correlation function plays a significant role in the detection/estimation process:

$$\boxed{\phi(\tau, f) \triangleq \int_{-\infty}^{\infty} \widetilde{f}\left(t + \frac{\tau}{2}\right) \widetilde{f}^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f}dt} \tag{30}$$

  primarily through its concomitant Ambiguity function $\theta(\tau, f) \triangleq |\phi(\tau, f)|^2$

  - To see this, note that the detection statistic L can be written as

$$L(\tau, f) = |\widetilde{y}(0)|^2 = |\phi(\tau, f)|^2 + ... \tag{31}$$

- The maximum-likelihood estimates of (τ_a, f_a) are given by

$$(\hat{\tau}_a, \hat{f}_a) = \text{argmax}_{\tau_0, f_d} L \tag{32}$$

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