2.163J/6.455J/12.518
Sonar, Radar and Seismic
Signal Processing
Fall 2006

Problem Set 2

MATLAB may be used for appropriate parts of the problem; however, most results with symbols representations need to sketched in with these labeled.

Issued: October 2, 2006
Due: October 16, 2006

Problem 1

We represent the transfer function in a simple waveguide as

$$H(f) \;=\; H_1(f) + H_2(f)$$

where

$$H_n(f) \;=\; \left[\begin{array}{ll} \frac{A_n}{\sqrt{R}} e^{-j\frac{2\pi}{c}\sqrt{f^2-(nf_o)^2}\,R} & f > nf_o \\ 0 & f \leq nf_o \end{array} \right.$$

with c the free space speed of propagation, R the range between transmitter and receiver, and the A_n are gain constants.

a) Determine the gain, the phase and group delays for each component, $H_n(f)$ of the overall transfer function, $H(f)$. Provide a *clearly labelled* sketch of the delays for each component.

b) The signal given by

$$x(t) \;=\; e^{-(Wt)^2} \cos(8\pi f_o t)$$

is the input to the system with $W << f_o$. Determine an approximate expression for the system output. (You may ignore dispersion effects.)

c) At what range are two components of the received signal separated in time by more than $1/W$, *i.e.* the two are resolvable in arrival time?

Problem 2

The environment has two reflectors as illustrated in the figure below

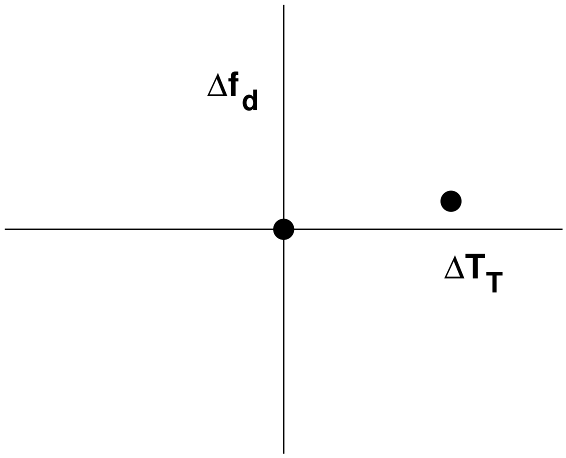

Figure 1: Two reflector model

The first reflector is located at $(0, 0)$ while the second is at $T_s, .1/T_s$ in the range-doppler plane. We use a gaussian LFM chirp for a signal given by [1]

$$\tilde{f} = \frac{1}{(2\pi T^2)^{1/4}}\exp(-(\frac{t}{2T})^2)\exp(j2\pi\mu\frac{t^2}{2})$$

where T is the signal duration and μ is the chirp rate. Assume the reflectors are of equal strength and a high SNR environment.

a) Consider the case without chirp. Sketch the range-doppler plane output for the cases $T << T_s$ and $T >> T_s$.

b) Now consider the case with chirp and $T \approx T_s$. Sketch the range doppler plane output for low values and large values of the chirp rate.

c) Discuss how the cases above pertain to range gating, *i.e.* resolving a target with large bandwidth, and doppler gating, *i.e.* resolving a target with a long duration.

[1]

$$\frac{1}{\sqrt{2\pi T^2}}\exp(-\frac{t^2}{2}) \leftarrow \mathcal{FT} \rightarrow \exp(-\frac{(2\pi fT)^2}{2})$$

Problem 3 Paired Pulses

A very simple signal for a sonar is given by

$$\tilde{f}(t) = \begin{cases} \frac{1}{\sqrt{2T}} e^{-j2\pi \frac{Mt}{T}}, & |t+NT| < \frac{T}{2} \\ \frac{1}{\sqrt{2T}} e^{j2\pi \frac{Mt}{T}}, & |t-NT| < \frac{T}{2} \\ 0, \text{elsewhere} \end{cases}$$

where T is a pulse width, M indicates the harmonic of $\pm 1/T$, and N a displacement to $\pm NT$. **Sketch the signal!!**

a) Determine the two frequency correlation $\phi_f(\Delta T_T, \Delta f_d)$ and $\Theta_f(\Delta T_T, \Delta f_d)$ along the principal axes $\Delta T_T = 0$ and $\Delta f_d = 0$. **(It is very useful to make a sketch of the functions one is integrating when determing limits.!)**

b) Discuss the choice of M and N *versus* the *a priori* knowledge of the range and doppler of a target.

Problem 4: Properties of ambiguity functions

1. Time and frequency invariance
Show that the ambiguity function is invariant to time shift and/or frequency shift, *i.e.* if

$$\tilde{f}_1(t) \;=\; \tilde{f}(t - T_x)$$

and/or

$$\tilde{f}_2(t) \;=\; \tilde{f}(t)e^{j2\pi f_x t}$$

then

$$\Theta_{f_1}(\Delta T_T, \Delta f_d) \;=\; \Theta_{f_2}(\Delta T_T, \Delta f_d) \;=\; \Theta_f(\Delta T_T, \Delta f_d)$$

2. Symmetry
Show that the ambiguity function has the following symmetry property:

$$\Theta_f(-\Delta T_T, -\Delta f_d) \;=\; \Theta_f(\Delta T_T, \Delta f_d)$$

3. Frequency domain evaluation
Sometimes it is easier to evaluate the two frequency correlation function and the ambiguity function in the frequency domain. Show that

$$\phi_f(\Delta T_T, \Delta f_d) \;=\; \int_{-\infty}^{\infty} \tilde{F}(\nu + \frac{\Delta f_d}{2})\tilde{F}^*(\nu - \frac{\Delta f_d}{2})e^{j2\pi \Delta T_T \nu} d\nu$$

4. Self transform
Show that Θ_f is its own Fourier transform, or

$$\int_{-\infty}^{\infty} \Theta_f(\Delta T_T, \Delta f_d)e^{-j2\pi(\Delta T_T \nu - \Delta f_d \tau)} d\Delta T_T d\Delta f_d \;=\; \Theta_f(\tau, \nu)$$

Problem 5: Hyperbolic frequency modulation

There is a large family of phase modulations which have been developed for radar and sonar signals. In this problem we explore hyperbolic frequency modulation which is used when for situations where sensitivity to wideband effects are a concern which is often the case in sonars. (See *Principles of High Resolution Radar*, A. W. Rihaczek, McGraw-Hill, 1969). The *real* transmitted signal is given by

$$s(t) = w(t)\sin(a\ln(1-kt)), \;\; 0 \le t \le T$$

where

$$\begin{aligned} k &= (f_{max} - f_{min})/(f_{max}T) \\ a &= -2\pi f_{min}/k \\ f_{min} &= \text{the initial frequency (Hz)} \\ f_{max} &= \text{the final frequency (Hz)} \\ T &= \text{signal duration (secs)} \\ w(t) &= \text{window function} \end{aligned}$$

We initially assume the window function, $w(t)$ is constant.

i) We define the instantaneous frequency as

$$f_{inst}(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

where $\phi(t)$ is the phase modulation. Find and plot $f_{inst}(t)$ for the case $f_{min} = 200\ Hz$ and $f_{max} = 300\ Hz$. Note that $f_{inst}(t)$ has the form of $-[2R/c - (1 - 2\dot{R}/c)t]^{-1}$ which cancels the time compression effect in a wideband signal. See the above reference for a complete discussion.

ii) Define the carrier frequency to be

$$f_c = \frac{f_{max} + f_{min}}{2}$$

Find the ambiguity function, $\Theta(\hat{T}, T_T; \hat{f}_d, f_{d_T})$ and plot it *versus* $\hat{T}, \hat{f}_d$. (This will require some computational assistance.) Compare the "chirp" ridge to a linear frequency modulated signal.

iii) Often a window is introduced to mitigate the effects of sidelobes as well as to avoid the impact of instantaneous loading, or step discontinuities, on the transmitter. One common window is the Tukey window which has the form

$$w(t) = \begin{bmatrix} \sin^2(\pi\frac{t}{2T_w}), & 0 \le t \le T_w \\ \\ 1, & T_w \le t \le T - T_w \\ \\ \sin^2(\pi\frac{t-(T-2T_w)}{2T_w}) & T - T_w \le t \le T \end{bmatrix}$$

Find and plot the ambiguity function for the windowed signal for $T_w = .25T$. This problem is indicative of a signal design typically used in current sonar systems.

Problem 6: Stepped frequency modulation

It is often convenient to use digital frequency synthesizers to generate frequency modulated (FM) signals because of their flexibility for programming the frequency sweep. This leads to "stepped FM" signals. In this problem we analyze a simplified version of a stepped LFM. The transmitted signal is given by

$$\tilde{f}(t) = \frac{1}{\sqrt{3T}} \left[\begin{array}{rrcl} e^{-j2\pi\Delta f t}, & -3T/2 & < t \leq & -T/2 \\ 1, & -T/2 & < t \leq & T/2 \\ e^{j2\pi\Delta f t}, & T/2 & < t \leq & 3T/2 \end{array} \right.$$

Assume that $\Delta f = \frac{n}{T}$ where n is an integer so that there are an integral number of cycles in each interval.

i) Find the ambiguity function along the zero range error axis, $i.e$ $\Theta(0, \Delta f)$.

ii) Find the ambiguity function at the following points along the zero doppler axis:

$$\Theta(T, 0)$$
$$\Theta(2T, 0)$$
$$\Theta(3T, 0).$$

iii) Just as with continuously swept FM the ambiguity function is also "sheared" with stepped FM. Determine the ambiguity function at the following points along the ridge with slope $\mu = \frac{\Delta f}{T}$:

$$\Theta(T, \Delta f)$$
$$\Theta(2T, 2\Delta f)$$
$$\Theta(3T, 3\Delta f)$$

What is value of the ambiguity function along the ridge $-\mu$, or $\Theta(mT, -m\Delta f)$ where m is an integer?

iv) Please discuss the generalization from the 3 digit pulse considered here to an N digit one. For the general case what are the range and doppler resolutions? Please include both the known and unknown doppler cases.