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Sonar, Radar and Seismic
Signal Processing

Problem Set 1

Issued: September 11, 2006
Due: September 27, 2006

Problem 1 - Correlation and matched filters

Many radar, sonar and geophysical systems use coded signals and "matched filters" for detecting, localization and/or communication. In this problem we explore some aspects of their implementation and use. Fig. 1a illustrates a transmitter which sends a signal $s(t)$, which is reflected off a target and observed at a receiver after a travel time T_d. In general, there is also noise, $n(t)$, present, but we do not include it in this analysis. If we also ignore attenuation affects, we can often model the signal as

$$r(t) = s(t - T_d)$$

The receiver consists of a matched filter which is a linear, time invariant system with an impulse response $h(t)$ given by

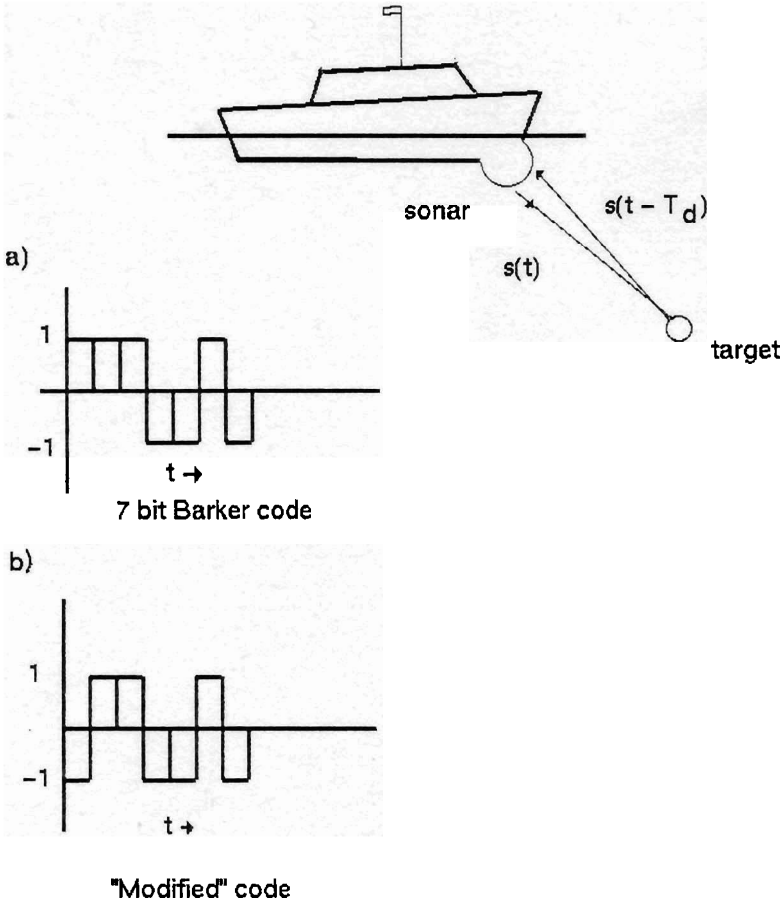
$$h(t) = s(-t)$$

a) Consider T_d to be zero initially. Find the receiver output and show that in general the receiver output is the autocorrelation of the signal $s(t)$. If T_d is non zero, describe an algorithm for estimating the travel time to the target, hence its range.

b) The impulse response is non causal, *i.e.* it is nonzero for $t < 0$. We can alternately implement the causal impulse response $h(t) = s(7T - t)$. How would one modify the range estimation algorithm?

c) In part (a) you presumably found an output signal which has a peak and some lower sidelobes. One of the objectives of good signal design is to find signals which have good correlation properties and low sidelobe levels since the narrow main lobe leads to accurate travel time estimates and the low sidelobes minimize the possibility that a noise spike leads to selecting the incorrect peak. The signal in part (a) is a member of a family called Barker codes which are often used because of their good correlation properties. Not all signals have such good properties. For example, consider reversing one of the "bits" in part (a) to form the signal in Fig. 1b. Find the matched filter response of this signal.

The Barker code is an example of a coded signal. The maximum length of such codes is 13 digits. Other codes signals such as shift register codes, or M sequences, are used in sonar and radar can have much longer lengths. For example such sequences have been used extensively in ocean acoustic tomography and radar astronomy.



Matched Filtering and Barker codes

Problem 2 - I/Q demodulation

One of the convenient practical ways of implementing a quadrature demodulator is to use gates which are clocked at $4x$ the carrier frequency of the narrowband signal. In Fig. 1 we illustrate such a system. The signal which gates the "I" component is a square wave on for the first quarter of a periodic signal operating with a period $T = 1/f_o$ while the gate for the "Q" component is on for the second quarter of the cycle. The pulses are T/4 long so the clocking needs to be at $4f_o$. The gated signals are then low pass filtered for the quadrature components.

a) Show that the system operates as a quadrature demodulator.

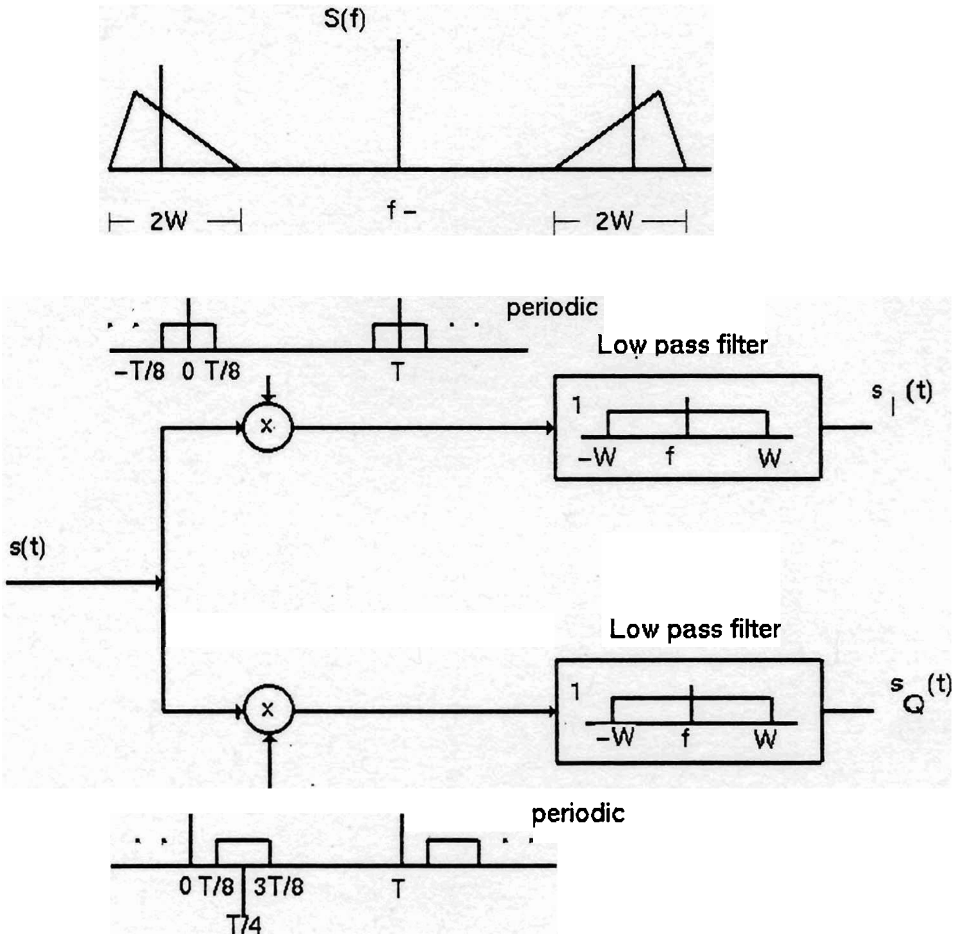
b) The signal $s(t)$ has the Fourier transform

$$S(f) = S_o(f - f_o) + S^*(-(f + f_o))$$

with

$$S_o(f) = \begin{cases} \frac{2}{3W}(f + W) & -W < f \leq W/2 \\ -\frac{2}{W}(f - W) & W/2 < f \leq W \\ 0 & \text{elsewhere} \end{cases}$$

Find $S_I(f)$ and $S_Q(f)$.



Quadrature demodulation

Problem 3 - Polar representations of Gaussian random variables

a) Let x_1, x_2 be independent, identically distributed, Gaussian random variables, $N(0, \sigma^2)$. Consider a polar representation of them given by

$$r = [x_1^2 + x_2^2]^{1/2}$$

$$\phi = tan^{-1}(x_1, x_2)$$

where $tan^{-1}(x, y)$ is the four quadrant arctangent function. Determine the joint density of r, ϕ. Find the marginal densities for r and ϕ.

b) Generalize the results to 3 dimensions where x_1, x_2, x_3 are each independent, identically distrubuted Gaussian random variables, $N(0, \sigma^2)$. Define

$$r = [x_1^2 + x_2^2 + x_3^2]^{1/2}$$

$$\theta = tan^{-1}(x_3, [x_1^2 + x_2^2]^{1/2})$$

$$\phi = tan^{-1}(x_1, x_2)$$

Find the joint density of r, θ, ϕ. Show that the variables r, θ, ϕ are statistically independent. Express $p_r(R)$ in term of a χ^2 density and interpret $p_\theta(\theta)$.

c) For the 2D case in part a consider the situation when the densities have means with

$$m_{x_1} = m\cos(\phi_m), m_{x_2} = m\sin(\phi_m)$$

Find the joint distribution for r, ϕ. Find the marginal densities for r and ϕ. This requires integrals which lead to modified Bessel functions, (The amplitude density is known as a Rician density and is useful when both coherent and incoherent signal components are present.)

d) Now let the phase ϕ_m be a uniformly distributed random variable over the interval $0 < \phi_m \leq 2\pi$. Determine the density $p_r(R)$.

These densities are quite important for analyzing the output of square law envelope detectors. Noise processes lead to the random components and the signal to the mean components. The averaging of the phase is from a channel model with a known amplitude but random phase which is usually result of positional uncertainty.

Problem 4 - Phase and group delay for surface gravity waves

a) The transfer function between two locations separated by R is given by

$$H(f) = \frac{A}{R} e^{-j(sgn(f))\frac{(2\pi f)^2}{g} R}$$

where g is the gravitational constant, R the separation and A a complex constant. ($sgn(x)$ is the sign function.) Determine the phase and group delays for an observer at R.

b) What are the constraints on range R and bandwidth W for dispersion to be small, *i.e.* a deviation from a constant group delay of less than $\pi/4$.

c) A wave generator introduces a narrowband signal at the source location of the form

$$x(t) = Ce^{-kt} * (t * \cos(2\pi f_o t) - sin(2\pi f_o t)) * U(t)$$

where f_o is the center frequency with $f_o >> k$ and C is a constant. Determine the complex envelope for $x(t)$ and its Fourier transform.

d) The narrowband signal of part C is applied to the ocean surface as modeled by $H(f)$ in part A. Determine an approximate expression for the output when dispersion can be neglected. Specialize the results to the case $((2\pi f_o)/g)R = 1001/2f_o$ and interpret the results.

e) Determine the dispersion factor for the gravity waves and determine the limits upon k in part C