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Sonar, Radar and Seismic Signal Processing

Review quiz

The following problems are indicative of the aspects of linear systems, Fourier transforms and probability which are expected as prerequisites for this subject. Please complete them; if you have difficulty, please contact me. These problems are for your information and will not be part of your grade.

1) Linear Systems - impulse response, convolution and correlation

A linear, time invariant system has an impulse response, $h(t)$, given by

$$h(t) = \begin{cases} \frac{1}{T} & |t - T_\circ| \leq \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$

An input signal $x(t)$ has the form

$$x(t) = \begin{cases} 1 & 0 < t < \frac{T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

a) Find and sketch the system output, $y(t)$.

b) Find and sketch the correlation function $R_y(\tau) = \int_{-\infty}^{\infty} y(t)y(t-\tau)dt$.

2) Fourier transforms

a) A signal $x(t)$ has the form

$$x(t) = e^{-at}U(t) + e^{-a(t-T_\circ)}U(t - T_\circ), \qquad T_\circ >> 1/a$$

($U(t)$ is the unit step function.) Find and sketch the Fourier transform, $X(f)$, of the signal.

b) The signal $x(t)$ is the input to an LTI system with transfer fucnction

$$H(f) = \frac{j2\pi f - a}{j2\pi f + a}, \qquad \text{for all } f$$

Find the energy spectrum $S_z(f) = |Z(f)|^2$ of the system output, $z(t)$.

3) Probability Theory

a) A random variable x has the probability density function

$$p_x(X) = \begin{cases} \frac{1}{L} & 0 \leq X < L \\ 0 & \text{elsewhere} \end{cases}$$

i) Find the mean and variance of the random variable, y,

$$y = \cos(\frac{2\pi X}{L})$$

ii) Find the probability density, $p_y(Y)$.

b) The random variables x_1 and x_2 are statistically independent and both have the same probability density $p_x(X)$ as described in part (a). Find the characteristic function, $M_d(jv) = E(e^{jvd})$ of the difference $d = x_1 - x_2$.