Subcritical scattering from buried elastic shells

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Buried objects have been largely undetectable by traditional high-frequency sonars due to their insignificant bottom penetration. Further, even a high grazing angle sonar approach is vastly limited by the coverage rate dictated by the finite water depth, making the detection and classification of buried objects using low frequency, subcritical sonar an interesting alternative. On the other hand, such a concept would require classification clues different from the traditional high-resolution imaging and shadows to maintain low false alarm rates. A potential alternative, even for buried targets, is classification based on the acoustic signatures of man-made elastic targets. However, the elastic responses of buried and proud targets are significantly different. The objective of this work is to identify, analyze, and explain some of the effects of the sediment and the proximity of the seabed interface on the scattering of sound from completely and partially buried elastic shells. The analysis was performed using focused array processing of data from the GOATS98 experiment carried out jointly by MIT and SACLANTCEN, and a new hybrid modeling capability combining a virtual source—or wave-field superposition—approach with an exact spectral integral representation of the Green's functions for a stratified ocean waveguide, incorporating all multiple scattering between the object and the seabed. Among the principal results is the demonstration of the significant role of structural circumferential waves in converting incident, evanescent waves into backscattered body waves, emanating to the receivers at supercritical grazing angles, in effect making the target appear closer to the sonar than predicted by traditional ray theory. © 2006 Acoustical Society of America. [DOI: 10.1121/1.2357711]

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I. INTRODUCTION

Buried objects have been largely undetectable by conventional high-frequency sonars, due to the low levels of energy penetrating into the sediment, in particular at subcritical insonification angles. Use of supercritical insonification has been limited by the small area coverage rate in shallow water. Critical grazing angles are typically $20^{\circ} - 30^{\circ}$, causing the range coverage of such a sonar to be limited to about twice the ocean depth. With subcritical penetration depth being inversely exponential with frequency,¹ a low-frequency sonar concept is therefore an attractive alternative. On the other hand, the reduced resolution at the lower frequencies makes traditional classification by imaging impossible, raising the need for alternative classification clues to keep the false alarm rates acceptable.

In this regard, the use of acoustic target signatures for classification may be a realistic alternative, in particular in the mid-frequency regime 1 < ka < 10, where the structural signatures are rich in information about target shape and composition, as demonstrated by decades of analysis of scattering by elastic shells, such as spheres and cylinders.^{2,3} For such acoustic signature classification, it is obviously critical that the relation between the target characteristics and the acoustic scattering is well understood. Resonance scattering theory² can be used to link both the position and the damping of the target resonances to the geometrical and physical properties of the elastic object, and some attempts have been made to set up a target recognition scheme based on the

analysis of the resonance spectrum. In these, and the studies that followed, the characterization of the scattering problem was mainly performed in the frequency domain.

Among the past theoretical solutions, exact formulations have emerged for calculating the scattered field from general elastic bodies embedded in a fluid half-space,^{4–7} in an elastic half-space,⁸ and in fluid layered waveguides.^{9,10} The work to follow was concentrated on the salient features of the scattering in the vicinity of medium boundaries^{11–13} and formed the basis for understanding the physics of acoustic interaction with objects that are either suspended in shallow water or buried.

Throughout the late 1980s authors such as Kargl and Marston¹⁴ have made observations and done modeling of the backscattering of short tone bursts from elastic spherical shells in terms of Lamb wave returns, axial reverberations, and glory effects often using a generalization of the geometric theory of diffraction to elastic objects in water described in Refs. 15 and 16. Thus far, theoretical analysis focusing on the scattering from elastic shells near boundaries has dealt with how the free-field resonance structure of the shell is influenced by the sediment loading and interface interactions. In particular, the discussion⁹⁻¹¹ was concentrated on the origin of modifications to the resonance structure giving rise to the classically observed symmetric and antisymmetric Lamb modes on shells and plates. Others¹⁷ have used resonance scattering theory (RST) models in combination with experimental results to address the dynamics of elastic waves

reradiated from spherical shells buried at different depths and insonified with a bounded beam at supercritical incident angles.

In mine countermeasures (MCMs) applications, resonance of man-made elastic targets is a pivotal concept that distinguishes them from rocks or other clutter that may have a similar minelike shape. Nevertheless, burial of an elastic target in the seabed results in a variety of modifications to the scattered response caused by different physical mechanisms, geometric constraints, and intrinsic properties of the sediment. While considerable attention has been devoted to the problem of scattering from submerged shells, and some attention to scattering from partially buried shells, flushburied shells have rarely been investigated, owing to the fact that flush-buried targets are difficult to detect and, even more so, the response they produce is challenging to analyze as the complete burial into sediment greatly affects the shell response observable in the water column.

While the initial response in the sonar detection community was to investigate the modes of energy coupling in the sediment through both the seabed roughness scattering and the frequency-selective phase matching from the ripple structure,¹ much of the attention is now being devoted to the analysis and the classification of the sediment altered elastic target scattering response. Being evident that lower frequencies possess better sediment penetration properties due to lower attenuation as well as a slower decaying evanescent field below the seabed interface, they become a preferred choice for detection of buried objects using subcritical incidence. In addition, it has been established that at lower frequencies, man-made targets, such as elastic shells, support the excitation and radiation of strong structural waves and resonances that create a specific acoustic signature that distinguishes them from other objects, and therefore can be used in detection and classification of targets.

In addition, knowledge of bistatic buried target responses may have potential in new multi-platform, autonomous surveillance systems. A leap from more traditional supercritical monostatic receiver-target configuration to a bistatic configuration is a logical step to obtain additional useful information for target classification.

On this background the motivation of the research presented here was to investigate the scattering from flush and partially buried targets in near-field bistatic configuration, insonified using subcritical incidence. To alleviate the shortcomings of the single-domain representation inherent in, e.g., the resonance scattering theory,² this work concentrates on a time-frequency representation rather than pure characterization of the scattering problem in the frequency domain. Furthermore, while ignored in the past, multiple scattering will be considered in the modeling and analysis, as it may play a significant role in complex shallow water environments where sound waves scatter from and between different objects and interfaces in close vicinity to each other.

This work aims to identify, analyze, and explain the fundamental effects of the sediment and the proximity of the seabed interface on the scattering of sound from elastic spherical shells insonified at low frequencies and subcritical incident angles. The global objective of this work is to develop an improved understanding of the fundamental physics of scattering from buried targets and subsequently to develop a robust methodology for their acoustic signature classification.

This paper describes the modeling and processing tools applied in the analysis, and then applies them to analyze and confirm a previously hypothesized dominance of elastic, circumferential waves in shaping the scattering from flushburied spherical shells. Presenting the newly developed target scattering model and a beamforming approach are as much in the center of interest of this work as is using those tools to confirm a hypothesis about a flexural Lamb wave phenomenon observed in the experimental data as well as in the model. Thus, a preliminary investigation of bistatic scattering from buried targets¹⁸ has postulated that at low frequencies (2-5 kHz range) and subcritical insonification, the target scattering is dominated by the specular scattering of the evanescent lateral wave, with the backscattering being excited by evanescent wave tunneling, similar to the behavior of a perfectly rigid target. In contrast, at high frequencies (10–15 kHz) the specular component becomes less significant due to the shallow penetration depth of the lateral wave, and instead a significant amount of energy is coupled into flexural, supersonic Lamb waves that radiate directly into the sediment and subsequently transmit energy into the water column at supercritical angles. This would suggest that the traditional plane-wave, ray-tracing approach to the propagation to and from the target is inadequate and should be replaced by a wave theory propagation model, adequately coupled to the target scattering model.

II. GOATS98 EXPERIMENT

A series of GOATS experiments was conducted with a long-term objective of developing new sonar concepts that exploit the information about target characteristics available in the 3-D multistatic field, and to attain improved detection and classification of buried targets.

The GOATS98 experiment was primarily aimed at developing an improved fundamental understanding of the physics of three-dimensional acoustic scattering from proud and buried objects. In addition, GOATS98 focused on exploring some of the fundamental aspects of the autonomous ocean sampling network technology for shallow and very shallow water MCMs. The GOATS98 experiment provided a first step towards the development of future MCM sonar concepts, by achieving a unique measurement of full threedimensional scattering by man-made and natural objects, along with the associated seabed reverberation. At the same time it demonstrated the use of AUVs as acoustic receiver platforms for MCMs, and their potential for rapid environmental assessment in shallow littoral environments.

Detection and classification potential of multi-static configurations was explored by investigating the differences in 3-D characteristics of seabed reverberation and target returns. Since both aspect-dependent targets and seabed ripples produce strongly anisotropic scattered fields, the differences in their spatial and temporal structure are better exploited by multi-static sonar configurations, with a potential for both

TARGETS AND ARRAY FIELD FOR GOATS 98



FIG. 1. Top view of GOATS98 experimental configuration. A stationary parametric source TOPAS parametric projector with a frequency range of 12-16 kHz was mounted on a 10-m-tall tower allowing target insonification at grazing angles below the critical grazing angle of 24° . The target field of interest consisted of three identical air-filled steel spherical shells S1 deeply buried, S2 flush buried, and S3 half buried. A 128-element HLA was placed 5 m vertically above the flush-buried target S2.

detection and classification performance enhancements compared to the more classical sonar systems.¹⁹ With the limited spatial coherence of target signals, the new multi-static sonar concepts exploit the differences in scattering directionality between targets and reverberation. The anisotropic spatial structure of the target field is critical for detection and classification enhancement of bi- and multi-static MCM concepts. In addition, the frequency dependence of monostatic and bistatic target scattering may be exploited for detection and classification of buried objects. Some of the previous modeling studies²⁰ have suggested that significant gains can be achieved by operating in a lower frequency regime (i.e., 1-3 kHz), where the evanescent penetration coupling is strong.

A. Experimental configuration

The GOATS98 experiment was carried out in May 1998 in 12–15-m-deep water off the coast of the island of Elba, Italy. Prior to the experiment a number of spherical and cylindrical targets were buried at different depths, within a 10 \times 10 m² area of sandy bottom. The thickness of the sand was several meters and it has provided an environment for the penetration and the target scattering investigation. A bistatic phase of the GOATS98 experiment consisted of a stationary parametric source (TOPAS) insonifying a target field. As depicted in Fig. 1, the parametric projector was mounted on a 10-m-tall tower that could be repositioned along a 20-m-long rail on the seabed, allowing target insonification at grazing angles below as well as above the critical grazing angle of 24° . The TOPAS transducer was used to insonify the target field with a highly directional beam. It consists of 24 horizontal staves, electronically controlled to form a beam in a selected direction. The secondary frequency range of the source was 2–16 kHz.

The target field consisted of three identical air-filled steel spherical shells S1 deeply buried, S2 flush buried, and S3 half buried, each of 1.06-m-o.d. and 3-cm shell thickness. The spheres were deployed in line with the TOPAS rail. The rationale behind having three identical spheres was to make it possible to evaluate the variation of detection of performance as a function of burial depth and grazing angle in monostatic, but primarily in bistatic, configuration. In addition, two steel cylinders were flush buried at aspect angles of 90° for C1 and 45° for C2. The 2-m-long and 50-cm-diam. cylinders were both water filled, having a shell thickness of 6 mm. Both of the cylinders were flush buried in the sediment. The different cylinder orientations effectively emphasized the fundamental difference in the bistatic target returns as well as providing a base for multiple aspect response analysis. The source transmitting a Ricker pulse with a frequency band of 2-16 kHz and the center frequency of

8 kHz insonified sphere S2 at a subcritical incidence of 18° using a highly directive beam. A combination of fixed and mobile arrays was available for recording the 3-D scattered field. Among them, a 128-element horizontal line array (HLA) in bistatic configuration with 9-cm element spacing was suspended 5 m vertically above the flush-buried spherical shell S2. The data received on the HLA proved to be of high quality and represents an important contribution towards establishing the validity of numerical models and investigating the above-stated target scattering physical phenomena.

B. Observations and hypotheses

In an effort to explain anomalous high-frequency scattering observed experimentally from flush-buried targets, an earlier paper¹⁸ presented a hypothesis regarding the role of elastic circumferential waves providing a strong mechanism for converting evanescent incident waves into propagating waves, which efficiently couple back into the water column. The findings of this preliminary study suggested that for subcritical insonification of buried targets, the traditional planewave, ray-tracing approach to the propagation to and from the targets is inadequate and must be replaced by full wave theory modeling. In the following, a more comprehensive analysis is performed, using experimental data and new modeling capabilities.

Thus, the GOATS98 experimental data set has been analyzed using a new focused beamforming approach, leading to identification of specular and elastic arrivals, and determining the different elevation angles at which they emerge from the seabed into the water column. As a result, this confirms the earlier hypothesis.

The target scattering scenario was modeled using the OASES-3D modeling framework, described later. As shown in Fig. 2, the TOPAS source was positioned on a 10-m-tall tower 29.5 m from the target S2, which was insonified at 18.7° grazing angle, well below the critical grazing angle of 24°. The AUV track was approximately perpendicular to the source-target axis, passing between the two at the distance 3.8 m from S2 at the point of closest approach. In this particular case a single scattering target modeling capability was employed. The resulting synthetic time series and spectrogram was obtained at 1-m spacing over a 10-m synthetic aperture. The time-frequency results of the above-described setup were quite unexpected and consisted of an initial strong low-pass-filtered specular response at 5 kHz, followed by a strong flexural response at a high frequency of 10 kHz, also followed by the flexural multiples at lower frequencies. While this result compares well to the actual AUV GOATS 98 experimental results, it is in contrast with the supercritical insonification results investigated by Tesei et al.¹⁷ that seem to follow the plane-wave ray-tracing theory and show a maximum in the flexural response at 8 kHz. Based on these results, the following physical mechanisms of subcritical scattering from flush-buried spherical shells were postulated. As depicted in Fig. 3, at low frequencies backscattering is dominated by the specular scattering of the evanescent, lateral wave. Therefore, for a subcritical receiver in the back-



FIG. 2. Layout of the GOATS98 bistatic imaging experiment. A TOPAS parametric source is mounted on a tower, which may be relocated along a horizontal rail to change angles of incidence on the seabed targets. A fixed 128-element horizontal hydrophone array was suspended 5 m over the targets for fixed bistatic measurements, while an AUV equipped with a receiving array and acquisition system was used as a moving receiver platform, creating synthetic apertures at different offsets from the target field.

scattering direction the evanescent wave is being excited by wave tunneling, with exponential decay in frequency. In contrast, as shown in Fig. 4, it is postulated that at higher frequencies the specular component becomes insignificant because of the shallow penetration depth of the lateral wave. However, for shallow burial depth the target curvature near the seabed allows the evanescent tail to couple efficiently into the flexural Lamb waves of which the supersonic component radiates into the sediment and transmits into the water column at supercritical angles. It therefore follows that the associated energy will arrive at water-column receivers at angles ranging from vertical, for a receiver above the target, to the critical angle at distant receivers.



FIG. 3. Low-frequency representation of subcritical scattering from a flushburied spherical shell. According to the hypothesis, the scattering at low frequency is dominated by the specular scattering of the evanescent lateral wave.



FIG. 4. High-frequency representation of subcritical scattering from a flushburied spherical shell. In contrast to the low frequencies, at higher frequencies the scattering is dominated by the A0 Lamb wave, which radiates into the sediment and transmits into the water column at supercritical angles.

Consequently, for subcritical insonification the specular arrival will be low-pass filtered relative to the incident field, while the flexural Lamb wave becomes high-pass filtered because of the more effective reradiation of the supersonic component into the sediment and back into the water column. In Sec. IV B 3 beamforming of the synthetic and experimental data was applied in order to test this hypothesis regarding subcritical insonification.

III. TARGET SCATTERING MODEL

A. Overview

The analysis of the scattering from buried targets is performed using a combination of array processing and the modeling using a new hybrid, wave theory framework (OASES-3D) combining a virtual source approach to target scattering with an established wave number integration model for the ocean waveguide propagation to and from the target.²¹ The virtual source approach, described in the following, is a generalization to arbitrary 3-D target geometry and composition of the classical wave-field superposition approach, e.g., the internal source density method, used by Stepanishen²² to model 3-D scattering from objects of revolution with Dirichlet or Neumann boundary conditions in an infinite, homogeneous fluid. This virtual source approach to target scattering can be considered a full 3-D generalization of virtual source approaches such as the one proposed by Veljkovic^{23,24} to model 2-D scattering from arbitrarily shaped objects near a seabed-water interface. All reported implementations of the method have had similar simplifications, which are avoided in OASES-3D. For example, Kessel²⁵ used a similar internal multipole expansion method in combination with a modal Green's function to model the scattering from objects in horizontally stratified waveguides, but again using ideal, homogeneous boundary conditions. Also, Kessel's, like Miller's,²⁶ approach does not allow the target to penetrate the waveguide interfaces and does not incorporate multiple scattering between the target and the adjacent boundaries. In addition, Sarkissian²⁷ did use a virtual source model to describe scattering by discrete objects in a waveguide, but the objects were not elastic and they were not

buried in the sediment, therefore simplifying the theoretical approach, eliminating the structural and a number of environmental effects. Different from Lim's⁷ approach, which employs the T-matrix formulation with multiple scattering, we develop and implement a different multiple scattering approach, which can handle realistic multiple scatterers of an arbitrary shape buried fully or partially in the sediment, as well as scattering from a nondiscrete entity such as a rough water-seabed interface. Additionally, in Lim's approach the bottom layer of the sediment is ignored, while we combine our scattering module with the OASES-3D wave-number integration approach, which fully incorporates the incident and scattered field interaction with all of the horizontally stratified layer interfaces.

B. Virtual source approach

The virtual source approach as implemented in OASES-3D involves three steps.²⁸ First, the incident field at the fully or partially buried target position in a stratified fluid-elastic waveguide is computed using standard wavenumber integration.²⁹ The scattered field is then represented by removing the target and replacing it by a distribution of virtual sources inside the volume occupied by the target. After superimposing the incident field with the virtual source field, the virtual source strengths are determined by satisfying the boundary conditions on the surface of the target. The boundary conditions for any elastic target may be expressed in terms of the dynamic stiffness matrix, expressing the unique relationship between the surface pressure and the normal displacement. As opposed to other coupling approaches such as the "scattering chamber" approach, the replacement of the target by its unique stiffness matrix does not require the treatment of the outer medium in the target model. Therefore, once the dynamic stiffness matrix for the target is determined, it can be used for arbitrary orientation and burial of the target. This characteristic of the approach makes the investigation of the sensitivity of the scattered field to the parameters, such as seabed properties, burial depth, and insonificaton geometry, exceedingly convenient.

Different from Kessel's²⁵ and Stepanishen's²² methods, this approach applies to general elastic objects with full 3-D geometry, requiring only a frequency-dependent stiffness matrix, associated with the target's internal structure and composition. In addition, the present approach allows the target to penetrate any interface in a horizontally stratified ocean environment, therefore providing a versatile numerical method for analysis of scattering from partially and fully buried targets. Furthermore, it takes into account multiple scattering effects within the target, as well as between the target and the environmental stratification as further investigated in Ref. 30.

The modeling of the target stiffness matrix is flexible and can be done using any target appropriate approach: a so-called "reverse" virtual source approach can be applied for a homogeneous fluid object, exact spherical harmonics representation can be used for spherical shells, or a more general numerical method such as finite elements can be used for other objects.



FIG. 5. Virtual source approach to scattering from partially buried targets in stratified ocean waveguides. The target is replaced by an internal, virtual source distribution generating a field in the background environment, which superimposed with the incident field, satisfies the boundary condition $\mathbf{p} = \mathbf{K}\mathbf{u}$, representing the target's dynamic stiffness properties.

The wave-field superposition principle is illustrated in Fig. 5. An arbitrarily shaped object in a stratified ocean is partially buried in the seabed. The stratification can include fluid as well as elastic layers, but it is assumed here for simplicity that the layers containing the target are isovelocity fluid media. The actual target is removed and replaced by a continuously stratified medium with a discrete distribution of N simple point sources, the unknown, complex strengths of which are represented by a dotted line. This source distribution is assumed to generate a field that is identical to the scattering produced by the target. Thus, if the surface of the target is discretized in N nodes, the total pressure \mathbf{p} and normal displacement \mathbf{u} are decomposed into the known incident field contribution \mathbf{p}_i , \mathbf{u}_i and the scattered field \mathbf{p}_s , \mathbf{u}_s :

$$\mathbf{p} = \mathbf{p}_i + \mathbf{p}_s, \quad \mathbf{u} = \mathbf{u}_i + \mathbf{u}_s. \tag{1}$$

The scattered field is generated by the virtual source distribution **s**:

$$\mathbf{p}_s = \mathbf{P}\mathbf{s}, \quad \mathbf{u}_s = \mathbf{U}\mathbf{s}, \tag{2}$$

with **P** and **U** representing $N \times N$ matrices containing the pressure and normal displacement Green's functions, respectively, between *N* virtual sources and *N* surface nodes.

The superimposed field on the virtual target surface must satisfy the boundary conditions associated with the real target. Thus, the field inside the true target must satisfy Green's theorem, providing a unique relation between the pressure and normal displacement on the surface. In a discrete representation with N surface nodes, this relation can be expressed in terms of a dynamic, frequency-dependent stiffness matrix **K**:

$$\mathbf{p} = \mathbf{K}\mathbf{u}.\tag{3}$$

Combining Eqs. (1)–(3) leads to the following matrix representation for the virtual source strengths:

$$\mathbf{s} = [\mathbf{P} - \mathbf{K}\mathbf{U}]^{-1}[\mathbf{K}\mathbf{u}_{\mathbf{i}} - \mathbf{p}_{\mathbf{i}}]. \tag{4}$$

The components of the stiffness matrix obtained using spherical harmonics representation are given in Ref. 31. Once the virtual source strengths are found from Eq. (4), the scattered field is obtained anywhere in the external medium by superposition, using the continuous medium Green's function, in this case the stratified ocean waveguide.

1. Green's functions

The Green's function for a stratified ocean may be computed using any approach valid in the vicinity of the source and at all angles. Here we apply the exact Fourier-Bessel wave-number integration formulation³² for stratified waveguides. Thus, the field produced by a horizontal distribution of sources can be expressed in an azimuthal Fourier series of the displacement potential $\phi(\mathbf{r}) = \phi(r, \theta, z)$,

$$p(r, \theta, z) = \phi_S + \phi_H$$
$$= \sum_{m=0}^{\infty} \left[\phi_S^m(r, z) + \phi_H^m(r, z) \right] \begin{cases} \cos m\theta \\ \sin m\theta \end{cases}, \tag{5}$$

where $\phi_S^m(r,z)$ and $\phi_H^m(r,z)$ are Fourier coefficients for the direct source contribution and the field produced by the boundary interactions, respectively. Both components are represented in terms of horizontal wave-number integrals,

$$\phi_{S}^{m}(r,\theta,z) = \frac{\epsilon_{m}}{4\pi} \int_{0}^{\infty} \left[\sum_{j=1}^{N} s_{j} \left\{ \cos m\theta_{j} \\ \sin m\theta_{j} \right\} \right] \\ \times J_{m}(k_{r}r_{j}) \frac{e^{jk_{r}|z-z_{j}|}}{jk_{z}} \left[k_{r}J_{m}(k_{r}r) dk_{r}, \right]$$
(6)

$$\phi_{H}^{m}(r,\theta,z) = \int_{0}^{\infty} \left[A_{m}^{+}(k_{r})e^{jk_{z}z} + A_{m}^{-}(k_{r})e^{-jk_{z}z} \right] k_{r} J_{m}(k_{r}r) \ dk_{r},$$
(7)

where k_r , k_z are the horizontal and vertical numbers, s_j is the complex source strength of source j at (r_j, θ_j, z_j) , and $A_m^+(k_r)$ and $A_m^-(k_r)$ are the complex azimuthal Fourier coefficients of the up-and-downgoing wave-field amplitudes produced by the multiple boundary interactions. They are found by matching the boundary conditions at all horizontal interfaces. ϵ_m is a factor that is 1 for m=0 and 2 otherwise.

Considering only the multiple scattering between the target and the seabed, all that is needed for generating the Green's function matrices in Eq. (4) is the two-half-space Green's function, which for the virtual source and receiver both being in the water column above the seabed becomes

$$G_{\omega}(\mathbf{r}_{i},\mathbf{r}_{j}) = s_{j} \frac{e^{ik_{1}|\mathbf{r}_{i}-\mathbf{r}_{j}|}}{4\pi|\mathbf{r}_{i}-\mathbf{r}_{j}|} + s_{j} \sum_{m=0}^{\infty} \left\{ \cos m\theta_{i} \cos m\theta_{j} \right\}$$
$$\times \frac{\epsilon_{m}}{4\pi} \int_{0}^{\infty} \left[J_{m}(k_{r}r_{j})R_{11}(k_{r}) \frac{e^{ik_{z}(1(z_{l}-z_{j}-z_{l}))}}{ik_{z;1}} \right]$$
$$\times k_{r} J_{m}(k_{r}r_{j}) dk_{r}, \qquad (8)$$

where R_{11} is the plane wave reflection coefficient for the seabed. Similarly, for the virtual receiver in the seabed, the Fourier-Bessel integral representation for the Green's function is



FIG. 6. Validation of spectral Green's functions. (a) Scattered acoustic pressure at 1 kHz 1 m from the center of the elastic sphere in an infinite fluid medium, with insonification at 45° . Solid curve shows the virtual source result using the spectral Green's function in. Eqs. (8) and (9). A dummy interface separating two identical water half-spaces passes through the center of the sphere. The result using the exact free-field Green's function is shown as a dashed curve. (b) Same, except lower half-space is air, and the dashed curve here shows the result obtained with the exact half-space Green's function using the method of images.

$$G_{\omega}(\mathbf{r}_{i},\mathbf{r}_{j}) = s_{j} \sum_{m=0}^{\infty} \left\{ \begin{aligned} \cos m\theta_{i} \cos m\theta_{j} \\ \sin m\theta_{i} \sin m\theta_{j} \end{aligned} \right\} \\ \times \frac{\epsilon_{m}}{4\pi} \int_{0}^{\infty} \left[J_{m}(k_{r}r_{j})T_{12}(k_{r}) \\ \times \frac{e^{i(k_{z;1}(z_{l}-z_{j})+k_{z;2}(z_{l}-z_{l}))}}{ik_{z;1}} \right] k_{r} J_{m}(k_{r}r_{i}) dk_{r}, \end{aligned}$$
(9)

with $T_{12}(k_r)$ being the transmission coefficient at horizontal wave number k_r .

C. Numerical implementation and validation

1. Virtual source distribution

Although the virtual source approach is, in principle, exact, the numerical stability of the solution is dependent on the virtual source distribution being such that it produces linearly-independent Green's function distributions over the surface nodes. In that regard it has been found empirically that a consistent convergence is achieved by distributing the surface nodes with a separation that is proportional to the local radii of curvature, and by placing a virtual source along the inward normal at each node, at a depth of approximately 0.6 times the node separation. This seems to provide the optimal compromise between diagonal dominance of the matrix to be inverted in Eq. (4) and efficient use of the dynamic range.

The numerical convergence of a virtual source scattering model has been validated by comparisons to exact spherical harmonic solutions for spherical shells²¹ and by comparison to full finite element solutions for general shapes.²⁸

2. Spectral Green's function

The computationally most intensive component of the present approach is the evaluation of the $N \times N$ pressure and displacement Green's function matrices **P** and **U** in Eq. (4) through the Fourier-Bessel representations in Eqs. (8) and (9). Here it is extremely important to take advantage of any

target symmetries and variables of limited dimension. Thus, for example, for targets with vertical axisymmetry, the virtual sources and surface nodes are naturally placed in "rings" at constant depth, thus heavily reducing the number of required values of Bessel functions. Other computational gains can be achieved by careful use of numerical devices such as precomputed tabulations of the exponential and Bessel functions, precomputing the wave-number functions, etc. Further gains are achieved by embedding the virtual source geometry within the integration kernels. In other words, instead of evaluating the spectral integrals for each source-receiver combination, the integral is performed simultaneously for all identical source-receiver combinations, leading to a reduction in computation order from $O(N^2)$ to O(N), with N being the number of the virtual sources and surface nodes.

Another key to the convergence and efficiency is the adherence to all the standard sampling guidelines for wavenumber integration,²⁹ including the use of complex integration contours, leading to very robust and accurate solutions as illustrated in Fig. 6. Thus, Fig. 6 compares the results of the virtual source approach using the spectral Green's functions in Eqs. (8) and (9) with the result obtained with exact Green's functions for an infinite medium with a dummy interface dividing the spherical target in two, and a homogeneous half-space with a pressure-release surface. Although these cases have very simple constant reflection and transmission coefficients of either unity or zero, the fact that a closed-form Green's function exists makes them ideal "sanity checks" for the spectral integrals. Figure 6(a) shows the scattered acoustic pressure at 1 kHz 1 m from center of an elastic sphere in infinite fluid medium, with insonification at 45°. The solid curve shows the virtual source result using the spectral Green's functions, while the result using the exact free-field Green's function is shown as a dashed curve. Figure 6(b) shows the comparison of the spectral and exact Green's function results when the lower half-space is replaced by air. Here the dashed curve shows the result obtained with the exact half-space Green's function computed using the method of images.

IV. ANALYSIS OF EXPERIMENTAL AND SYNTHETIC DATA

The analysis is performed using parallel, identical processing of the experimental data and the associated synthetics generated using the OASES-3D modeling framework. The analysis included time-frequency processing of the real and synthetic time series, as described in Sec. IV B 2. The array processing of both data sets are compared in Sec. IV B 3. In addition to providing a physical explanation of the observed behavior, the analysis verifies and validates the models.

A. Tools and approaches

1. Arrival time analysis

The first step in the analysis is the characterization of the dispersion of compressional S0 and flexural A0 waves as they revolve around the shell. Tesei¹⁷ has completed a comprehensive analysis of the dispersion of the structural response of the spherical shells applied in the GOATS98 experiment.

Using the results of this analysis, a travel-time tool was developed to facilitate quick calculation of expected arrival times of specular, A0, and S0 waves from a point on the shell to a particular point in the water column.

Using 3-D vector calculus, the travel-time modeling tool determines the points on the shell from which A0 and S0 waves emanate to any point in the water column. Given known locations of the HLA receivers, it calculates the time it takes for the specular and the elastic returns to reach a given receiver. Therefore, corresponding travel times in the shell, the sediment, and the water are obtained. Snell's law is used to calculate the refraction effects at the water-sediment interface.

The procedure is carriedout as follows. Knowing the point at which the specular reflection is located on the shell, the distance traveled along the shell in both clockwise and counter-clockwise directions is calculated. The angle of radiation from each waveform is known by utilizing the phase matching formula given by

$$\theta_c^* = \arcsin\left(\frac{c_{\text{ext}}}{c_{\text{shell}}}\right),\tag{10}$$

where c_{shell}^* the shell membrane wave speed and c_{ext} is the shell exterior medium speed. The point of the emanation on the shell can be obtained by a dot product between the vector path in the sediment and the shell radial vector. The point at which the ray hits the water-seabed interface is determined by the location of the receiver and the angle obtained by Snell's law.

Once the distances connecting the points of interest are determined, the corresponding wave speeds are necessary to determine the times of travel. In the study conducted by Tesei *et al.*,¹⁷ an auto-regressive spectral method was used to determine the phase and group speeds of compressional S0 and flexural A0 waves for a sediment- and water-loaded elastic shell. Dispersion curves of S0 and A0 waves give frequency-dependent speeds in the 2-15 kHz regime, which coincides with the frequency regime of the GOATS98 ex-

periment. For the purpose of the travel-time calculation for the expected times of specular and flexural wave arrivals, speeds of A0 and S0 waves at the center frequency of 8 kHz are calculated from the group speed dispersion curves provided by Tesei,¹⁷ with a result of 2200 m/s for the A0 wave group speed and 5650 m/s for the S0 wave group speed, when the shell was flush loaded in sediment. Once the speeds of elastic waves are established, and the compressional wave speeds in the sediment and water were determined *in situ*¹ to be 1640 and 1520 m/s correspondingly, the travel time calculation of particular arrivals can be carried out. In addition, it was also established¹ that the sediment attenuation was $0.5 \text{ dB}/\lambda$. The times of travel in shell, sediment, and water for clockwise and counter-clockwise A0 and S0 waveforms and their multiples can be used to identify the arrivals in time-frequency plots as it was done in Sec. IV B 2.

In the sections to follow, these theoretical expected arrival times are superimposed on the target scattering model results as well as on target scattering experimental results. By carrying out this procedure, the waveforms that comprise the target elastic response are identified in time, and the peculiarities of the amplitude and the frequency contents of identified arrivals can then be attributed to a particular waveform and further analyzed.

2. Focused beamforming approach

In addition to the time of arrival analysis, the following analysis was also done on a 1–46 receiver subarray, positioned in the backscattering direction with respect to the flush-buried target S2, as shown in Fig. 14. By focusing the beamformer to different points on the seabed, the range and the elevation of different wave types were determined, therefore determining the effective radiation strength of the particular area of the seabed. In order to obtain the information of interest, focused beamforming was repeated for each discrete elevation ϕ and the ranging azimuth θ . Therefore, a matrix that contained arrivals at different elevation-azimuth point pairs was generated.

The next stage consisted of projecting the elevationazimuth pairs to the seabed interface. Since the positions of the subarray sensors and the position of the flush-buried target was known, an intersection of the beamforming cone and the plane containing the seabed producing a hyperbola had to be calculated to obtain a set of XY points on the seabed surface. For this purpose, as shown in Fig. 7, a new coordinate system is introduced where the array is temporarily aligned with the y axis on the ground, and thus the following expression for the equation of the cone and the equation of the seabed are obtained:

$$\frac{x^2}{a^2} + \frac{(z - z_0)^2}{a^2} - \frac{y^2}{b^2} = 0, \quad z = 0.$$
 (11)

At the intersection of the cone and the seabed the following equation is obtained

$$x^{2} + z_{0}^{2} - (\tan^{2} \alpha)y^{2} = 0, \qquad (12)$$

where $\tan \alpha = a/b$ and $\alpha = \phi - \pi/2$, giving



FIG. 7. Focused beamforming, projection of elevation, and azimuth angle on to an (X, Y) point on the seabed.

$$r = \frac{z_0}{\sqrt{\tan^2 \alpha \sin^2 \theta - \cos^2 \theta}}$$
(13)

from which the desired coordinate point pairs $(\overline{X}, \overline{Y})$ for each α and θ are readily available. A rotation needs to be introduced to obtain the actual seabed coordinates (X, Y) that correspond to the GOATS98 experimental configuration:

$$X = \overline{X}\cos\theta_0 - \overline{Y}\sin\theta_0,\tag{14}$$

$$Y = \overline{X}\sin\theta_0 - \overline{Y}\cos\theta_0,\tag{15}$$

where θ_0 is the angle of azimuthal rotation of the HLA with respect to the y axis. This procedure was repeated for varying ϕ and θ angles, giving a family of hyperbole on the seabed comprised of different XY pairs. Figure 8 shows the XY pairs, on the seabed marked with dots, for ranging θ and ϕ . For each elevation and a range of azimuth angles, the intersection is a single hyperbola in the XY plane. As the elevation angle ϕ increases, different hyperbole describe a map of points on the seabed.

The final stage consisted of properly delaying the matrix of focus beamformed results, creating a real-time map of beamformed seabed radiation strength caused by the elastic shell response beneath it. The matrix, up to that point, contained beamformed results that represent repetition of a fixed set of events at different elevation angles used for beamforming. However, what was wanted for the projection on the seabed was a sequence of snapshots of the radiation strength of the seabed surface in real time, that is, an actual beamformed response of the effect of the radiating elastic target producing first the specular and then the set of elastic waves that hit the seabed surface at a certain elevation and azimuth in real time.

As shown in Fig. 9, to obtain this real-time map of seabed radiation strength, beamformed arrivals at each (X, Y)coordinate pair were shifted in time by Δt_i , which is the time necessary for the return to travel through water from the point of emanation on the seabed, (X_i, Y_i) , to the center of the subarray:



FIG. 8. Focused beamforming target field. The dots represent the calculated XY point pairs that comprise a family of hyperbole generated by the intersection of the beamforming cone and the seabed for varying elevation and azimuth.



FIG. 9. In order to obtain the real-time map of seabed radiation strength, beam-formed arrivals at each (X, Y) coordinate pair were shifted in time by Δt_i , which is the time it takes for the return to travel through water from the point of emanation on the seabed (X_i, Y_i) to the center of the subarray.

$$\Delta t_i = \frac{R_{X_i, Y_i}}{c_{\omega}}.$$
(16)

As a result of this novel focus beamforming approach, various types of specular and elastic arrivals were identified, demonstrating the different elevation angles at which they emerge from the seabed into the water column.

B. Subcritical scattering by the flush-buried sphere— Results and discussion

1. Multiple scattering effects

One of the central objectives of this work is to analyze the mechanisms of scattering from buried targets under subcritical insonification, distinguish them from the ones generated using supercritical insonifications, and identify the target signature features that can be used to enhance detection and classification of buried mines. While multiple scattering has often not been taken into account in supercritical scattering from targets, it is likely to play a more significant role here, and it may become a possible classification clue for flush-buried target signatures for targets insonified using evanescent insonification. One of the goals in the study was to identify the features in the elastic shell scattering response brought about by multiple scattering, and use models that can incorporate it, in order to be able to compare the experimental and model data.

The snapshots of the results shown in Fig. 10 illustrate the effects of multiple scattering while also showing the peculiarities of the evanescent incidence on a flush-buried elastic spherical shell generated using the numerical target scattering module described in Sec. III that incorporates multiple scattering. The input parameters used for model runs were as follows: the incident field frequency was 3 kHz, the radius of the 3-cm-thick, 7700-kg/m³ dense steel shell was 0.503 m, the shell compressional wave speed was 5950 m/s, and the shell shear wave speed was 3240 m/s. The compressional speeds of sand and water were 1640 and 1520 m/s, respectively, with the sand having 0.5 dB/ λ attenuation. On the left-hand side, a plane wave is supercritically incident at 35°



FIG. 10. Propagating (left) versus evanescent (right) incidence on a flushburied spherical shell—multiple scattering included. The difference between the propagating and the evanescent incidence on a flush-buried elastic spherical shell was shown. The result was generated using the multiple scattering numerical model described in Sec. III. The figure also illustrates the contribution of multiple scattering to the flush-buried target response, and it is most obvious in the snapshots in the third and the fourth rows, where we can see the multiple reflections of the specular of the target hitting the water seabed interface, coming back to the target, and being reflected from it again towards the sediment-water interface.

on the flush-buried shell. The pronounced specular response of the target is visible, followed by the clockwise and counter-clockwise S0 compressional elastic waves, and the A0 flexural elastic wave. It is worth noting that clockwise and counter-clockwise components of the elastic waves are of similar amplitudes. The fourth row illustrates the effect of the clockwise and counter-clockwise components of the flexural A0 wave meeting and interacting after each one has traveled a full revolution around the shell. On the right-hand side, a plane wave is incident subcritically on the seabed, producing an evanescent field that interacts with the flush-buried shell. In this case the response is quite different. The shell specular return is less pronounced, with the strongest response caused by the evanescent field coupling with the shell and the A0 wave circumnavigating the shell in the clockwise direction. The compressional S0 responses are apparent as yellow spots around the shell radiating as they travel faster than the A0 waves. The clockwise part of A0 response is shown here to be considerably stronger than its counter-clockwise counterpart, owing to the nature of the insonifying evanescent field that exponentially decays in depth.

This particular result is related to the Lamb wave hypothesis discussed in Sec. II B. It is the pronounced A0 Lamb wave, which according to the hypothesis, revolves around the target, radiates into the sediment, and transmits into the water column at supercritical angles.

Figure 10 also illustrates the contribution of multiple scattering to the flush-buried target response. This manifestation is the most obvious in the third and fourth row snapshots, where we can see the multiple reflections of the specular of the target hitting the water seabed interface, coming back to the target, and being reflected from it again towards the sediment-water interface. While the phenomenon is the most obvious in the propagating case on the left, due to the higher strength of the transmitted field, it is also visible in the evanescent incident case on the right. The multiple reflections of the flexural A0 and compressional S0 waves are also visible in the following rows.

2. Time-frequency analysis of the flush-buried shell

When a plane wave is incident on a shell in free space, the clockwise and the counter-clockwise returns travel the same path to the receiver in a monostatic configuration. As they revolve around the target, the flexural A0 returns radiate at a phase matching angle θ_c^* almost perpendicular to the target radius. The compressional S0 waves radiate, in turn, at a phase matching angle θ_c^* that is smaller than the A0 angle.

However, when the shell is flush buried in the sediment, and insonified at subcritical angles, an evanescent field is created in the sediment. For evanescent insonification, and the HLA receiver in bi-static configuration, such as the case in the GOATS98 experiment, Fig. 11 shows that when the time it takes to travel around the shell, through the sediment, and through the water column is added, A0I arrives at the receiver location before A0II. The same applies to S0I and S0II arrivals as well.

An out-of-plane source-receiver configuration (i.e., HLA receivers that are azimuthally away from the source-target axis in Fig. 1) is considered in Fig. 12. It represents the time series and the spectrogram of the GOATS98 experiment for a flush-buried target and receiver 26, located out of plane in the backscattering direction. The significance of the out-of-plane receivers in bistatic configuration is that they measure the 3-D target scattered field, supplying the classification and detection information, which a traditional monostatic in-plane receiver configuration¹⁷ is not capable of providing.



FIG. 11. Schematic of an evanescent field incident on a flush-buried elastic shell. For evanescent insonification and the HLA receiver in bi-static configuration, when the time it takes to travel around the shell, through the sediment, and through the water column is added, A0I arrives at the receiver location before A0II.

The locations of the arrivals on the spectrogram point towards the fact that the time of arrival calculations are accurate, as the specular and elastic arrivals coincide with the calculated expected times marked with black and green lines and calculated using the arrival time method from Sec. IV A 1. The spectrograms in Figs. 12 and 13 were both calculated using discrete Fourier transforms of Hanning windowed, well-overlapping segments of the corresponding time series. The number of FFT points used was 1024 giving the frequency resolution of 50 Hz, while the window length was 0.16 ms with the overlap of 0.12 ms, making the time resolution of the result about 0.1 ms. Due to the particular wave speeds and the configuration, SOI arrives before the specular, which is centered around 8 kHz. SOII is of considerably high-frequency content as it arrives at 12 kHz, and the following S0Ix2 at 10 kHz. Close to 23.5 ms, an A0I waveform arrives at a high frequency of 10 kHz, which is significantly higher than the expected center frequency of 8 kHz. At 24 ms A0II arrives at significantly high 12 kHz center frequency. The fact that SOI, AOI, and AOII have higher frequency content than the expected 8 kHz center frequency to begin with, provides evidence for the hypothesis from Sec. II B, where it is suggested that, under subcritical insonification, the flexural Lamb wave becomes high-pass filtered because of the more effective reradiation of the supersonic component into the sediment and the water column. By extension, the observed high frequency of the compressional part of the response can potentially also be affected by the more efficient coupling of the incident evanescent wave. It is believed that this is the first time in literature that the highpass filter anomaly is demonstrated using experimental data under these conditions. It should be also noted that, unlike what is observed for propagating incidence,¹⁷ the magnitude of the A0II flexural response is 8 dB higher than that of the specular response for the highly out-of-plane receivers in the backscattering direction.

Both the frequency content and the amplitude content differences described in the above paragraph present new



FIG. 12. Time series and spectrogram for a flush-buried target—experimental data, receiver 26. Calculated time of arrival expectations for S0 arrivals are marked with vertical green lines, while the expectations of A0 arrivals are marked with black lines. SOII is of considerably high-frequency content as it arrives at 12 kHz. Close to 24 ms, a strong A0II waveform arrives at a high frequency of 12 kHz, which is significantly higher that the expected center frequency of 8 kHz. Therefore, the high-pass filter anomaly of the flexural target response is identifiable in the experimental data. Also, the magnitude of the A0II flexural response is 8 dB higher than that of the specular response.

identification points for bistatic classification of buried targets insonified using evanescent incident fields.

Figure 13 shows the model result in the form of the time series and the spectrogram for the flush-buried target and the out-of-plane receiver 26. As before, S0I arrives before the shell specular return due to the target-receiver configuration. This agrees well with the corresponding experimental result in Fig. 12. The obvious high amplitude and 8 kHz center frequency specular is followed by SOII arrival at 7 kHz. This is lower than the corresponding experimental result in Fig. 12 that arrives at 12 KHz. Here, and for the rest of the time series record, experimental data consistently assign higher frequency contribution throughout the various wave types of the elastic response than the corresponding model results, even though the sediment attenuation used in the model corresponds to the measured value in an elaborate study by Maguer.¹ A few possible explanations of these discrepancies to be investigated in the future are shear and porosity effects not included in the fluid sediment model. Thus, energy is radiated into the shear waves, which are much slower than the flexural wave field.

Going down the time axis, the combined S0Ix2 and A0I effect is of significantly smaller amplitude and lower frequency content than that shown in the experimental results in Fig. 12. Also, the distinctive A0II flexural arrival dominates the elastic response, and it arrives at 9 kHz frequency, which is lower than in the in-plane receiver case but still higher than the 8 kHz center frequency traditionally observed with supercritical insonification and monostatic configuration. Yet again, this time for the modeled result, it is shown that the AOII flexural return arrives at higher frequencies than the corresponding specular return, validating the flexural Lamb wave high-pass filtering hypothesis. After the following several multiples of S0 returns, a repetition of weak AOIx2 arrivals and a strong AOIx2 flexural arrivals is observed.

Concerning the timing of the arrivals in the model data in Fig. 13, while the dominant AOII arrivals coincide with the black line predictions, SOIx2 and AOI are offset from their prediction times by 0.1 and -.0.1 ms, respectively. On the other hand, those same expected times of arrival for SOIx, AOI, and AOII agree well for the experimental data in Fig. 12. The fact that in Fig. 12 virtually all of the calculated expected times of arrival coincide with the experimental data, but are in some instances offset from model data in Fig. 13, might suggest some small inconsistencies in the timing of arrivals generated using the model. Additionally, the model result overall does have a more synthetic, clean quality to it, compared to the experimental response, which seems to be noisier.

In summary, the basic characteristic observed for a bistatic receiver for a flush-buried target under evanescent insonification confirms the high-pass filtering Lamb wave hypothesis. In addition, they present new identification points for bistatic classification of buried targets by pointing at the



FIG. 13. Time series and spectrogram for a flush-buried target—model data, receiver 26. Calculated time of arrival expectations for S0 arrivals are marked with vertical green lines, while the expectations of A0 arrivals are marked with black lines. The combined S0Ix2 and A0I effect is of smaller amplitude and lower frequency content than that shown in the experimental results in Fig. 12. Also, the distinctive A0II flexural arrival dominates the elastic response, and it arrives at 9 kHz frequency.

amplitude and frequency differences between the specular, the clockwise, and the counter-clockwise components of the flexural waves as further discussed in Ref. 30.

3. Focused beamforming results

Array processing was applied to the HLA experimental and model data, to establish the elevation angle ϕ at which the specular and the elastic returns emerge from the seabed into the water column. This particular angle would provide information about the sediment propagation paths of the scattered wave types, and therefore gives insight in the physical phenomena that take place around the radiating sediment loaded shell.

Since the HLA was operated in a very near field with respect to the S2 target, focused wideband beamforming along sections of the HLA was initially done to determine the azimuthal angle of arrival of returns. The near field is defined as $r \leq L^2/\lambda$, where in our case the distance to the target is r=5 m, and at the center frequency of 8 kHz, the wavelength $\lambda = 0.19$ m, with the length L of the whole array aperture of 11.43 m. Also, the HLA was segmented in three subarrays in order for the beamformer to be able to handle the wave-field inhomogeneities, but at the expense of the focusing ability. Figure 14 represents the configuration of a backscattering segment of the array, receivers 1-46, in its coordinate system where the center of the coordinate system is lying on the seabed, ϕ is the elevation angle with respect to the vertical z axis, and θ is the azimuth angle with respect to the x axis. At a range R, the beamforming cone intersects the seabed, creating a set of intersection points that describe a hyperbola.

As we are about to see, a fairly conventional practice of implementing a fixed-elevation focused beamformer, which has been implemented by other authors to experimental data in monostatic configuration,³³ ends up being marginally useful for our purposes, and therefore a new type of a beamforming approach was here developed in Sec. IV A 2 to extract more meaningful information about the physical mechanisms occurring during the elastic shell radiation process.



FIG. 14. Focused beamforming configuration of a back-scattering segment of the array, receivers 1–46, in the focused beamforming coordinate system where the center of the coordinate system is lying on the seabed, ϕ is the elevation angle with respect to the vertical *z* axis, and θ is the azimuth angle with respect to the *x* axis. At a range *R*, the beamforming cone intersects the seabed, creating a set of intersection points that describe a hyperbola. Note: The spherical target is flush buried in the sediment.



FIG. 15. Focused beamforming, flush-buried target—experimental data. This figure represents a part of a filmstrip of a 3-D representation of the location *XY* at which different beamformed target responses and their multiples emerge, and the strength of the response at that particular point is represented along the *Z* axis. The *XY* location of the flush-buried target is marked with a white vertical arrow, and the real time of the radiating shell response is marked in the upper right corner of each snapshot. Note that the arrow designates the target location, not the emerging wave location. The left snapshot shows that at 21.12 ms the specular arrival emerges at a seabed spot that corresponds to the coordinate (*X*, *Y*) pair of (-0.5,0) m. The physical significance of this result is related to the fact that this particular point on the seabed corresponds to a particular elevation and azimuth angle pair (ϕ_{spec} , θ_{spec}) at which the specular response is scattered from the buried elastic shell S2. In the right snapshot the A0II return is shown to arrive at a seabed point (*X*, *Y*)=(-0.8,0) m, which is different from the specular, pointing at a different elevation angle of the elastic waves propagation through the sediment and coupling into the water column than the specular target return.

4. Experiment beamforming results

Figure 15 represents filmstrips of a 3-D representation of the beamformed scattering process from the flush-buried sphere S2 that was captured in time as snapshots of specular and elastic waveforms radiating from different points on the seabed. X and Y coordinates on the seabed are obtained from ϕ and θ as described in the previous section, and the strength of the response at that particular point is represented along the Z axis. The XY location of the flush buried target is marked with a white vertical arrow, and the real time of the radiating shell response is marked in the upper-right-hand corner of each snapshot.

In Fig. 15 the left snapshot shows that at 21.12 ms the specular arrival emerges at a seabed spot that corresponds to the coordinate (X, Y) pair of (-0.5, 0) m. The physical significance of this result is related to the fact that this particular point on the seabed corresponds to a particular elevation and azimuth angle pair $(\phi_{\text{spec}}, \theta_{\text{spec}})$ at which the specular response is scattered from the buried elastic shell S2. In the right snapshot we can see the radiation strength of the sand for the stronger A0II, which emanates from the seabed into the water column at a different point that corresponds to a new (X, Y) pair of (-0.8, 0) m.

To more precisely determine the angles of arrival of specular and elastic returns, Fig. 16 was created using filmstrip snapshots of beamforming results with the elevation angle ϕ on the X axes and the azimuth angle θ on the Y axes. It shows at what particular elevation and azimuth angles the arrivals emerge from the seabed into the water column. The purple circle on the snapshots marks the (ϕ, θ) location of the target with respect to the subarray center. Specular, compressional S0, and flexural A0 waves and their multiples are identified on different time snapshots of the figure, where time is shown in the upper right hand corner. The first snapshot shows that the shell specular arrives at the sediment at an elevation ϕ of 143° and the azimuthal angle of 80°. The reader could refer to Fig. 14 for the graphical definition of the HLA-target elevation angle ϕ and the azimuthal angle θ . The next snapshot on the right shows that the shell compressional return SOII arrives at roughly the same azimuth but very different elevation, namely $\phi = 150^{\circ}$. It should be noted that the higher elevation angle from the HLA to a point on the sediment surface means a shallower elevation angle from the target to that point of waveform emanation on the sediment surface. Therefore, this snapshot shows that the elastic shell compressional wave travels through the sediment at a 7° shallower angle than the target specular return.

The next snapshot shows the elevation of the S0Ix2 wave, which arrives at $\phi = 151^{\circ}$ and is also shallower than the specular arrival. The next snapshot on the right shows that the elevation angle of the first flexural AOI arrival is 147°, which also makes it shallower than the specular arrival. In the following snapshot the elevation angle of the flexural A0II arrival is 146°. The last snapshot shows the second revolution of the AOII return, AOIIx2, coming in at a distinctly shallow 161°. A possible explanation for A0Ix2 emerging at a different point than A0I follows from a sequence of physical mechanisms taking place during the target scattering process: longer distance circumnavigated, in form of revolutions, causes more attenuation to the waveforms, higher frequencies attenuate faster than the low ones, a change in frequency translates to a change in velocity governed by dispersion curves, a change in velocity means that the waveforms emerge at a different angle θ_c^* to the shell as demonstrated by the phase matching equation (10), which in turn causes the arrivals to hit the seabed at a different point and therefore at a different elevation angle. In addition, the overall resolution of the beamformed results is about 3° depending on the wave type, while the differences in the elevation angle between the specular and the first set of elastic arrivals are 7° and 4°, respectively, confirming that the beamforming resolution is high enough to support the conclusions.

Using this novel type of focused beamforming, the actual at-sea experimental results give quantitative base for the hypothesis about the physics of the propagation of elastic



FIG. 16. Elevation-azimuth focused beamformed results—experimental data. This figure shows at what particular elevation and azimuth angles the targetradiated arrivals emerge from the seabed into the water column. The purple circle on the snapshots marks the (ϕ, θ) location of the target with respect to the subarray center. Specular, compressional S0, and flexural A0 waves and their multiples are identified on different time snapshots of the figure. The first snapshot on the left shows that the shell specular arrives at the sediment at an elevation ϕ of 143° and the azimuthal angle of 80°. The next snapshot on the right shows that the shell compressional return S0II arrives at roughly the same azimuth but different elevation, namely ϕ =150°, a 7° shallower angle than the target specular return. The figure also shows that the elevation angle of the first flexural A0I arrival is 147° and A0II is 146°, which makes the flexural waves also shallower than the specular target return. In the last snapshot the second revolution of the A0II return, A0IIx2, comes in considerably shallow at 161°.

waves under evanescent insonification described in Sec. II B, and make currently used wave-tracing arguments that assume all of the wave paths as to and from the target, unsound. The results and the analysis presented here differ from those presented by Lim⁷ in the fact that he shows the angular dependence of the entire compound backscattered response, but does not assign the angular dependence of the individual emerging wavetypes such as specular, compressional, and flexural waves. He seems to be mostly concentrated on the differences in the compound backscattered field between the attenuating and nonattenuating sediment cases.

In addition, the result presented here confirms experimentally the proposed potential of yet another way of using structural waves to aid in the buried target classification, rather than the currently commonly used specular response alone, which often carries only a limited amount of useful information.

5. Model beamforming results

The whole multiple elevation beamforming procedure described in Sec. IV A 2 was repeated on the model data. In Fig. 17 the left snapshot shows that at 24.9414 ms the specular arrival emerges at a seabed spot whose center corresponds to the coordinate (X, Y) pair of (-2, -0.2) m. On the right, the seabed radiation strength due to the stronger AOII return is seen to arrive at (X, Y)=(-1.6, 0) m at its own azimuth-elevation pair $(\phi_{AOII}, \theta_{AOII})$ characterized by the spe-

cific physics of radiation around the shell and propagation through the sediment. This process is continued in time for the elastic responses that follow. The full sequence of film strips showing the emanation of the specular and all of the compressional and flexural responses can be found in Lucifredi's dissertation.³⁰

The physical significance of the results shown above lies in the fact that they unambiguously show that the elevation angles of specular and elastic arrivals at which they emerge from the seabed into the water column are different, and therefore travel to the seabed-water interface at different elevation angles, a realization that is contrary to the traditional wave-tracing argument and in agreement with the hypothesis described in Sec. II B. In predicting this phenomenon, the virtual source model results are in agreement with the experimental results described in the previous paragraphs. Nonetheless, the differences between the experimental and model results do exist and are further addressed below.

In order to further investigate the phenomenon and, more precisely, determine the angles of arrival of specular and elastic returns, Fig. 18 was created using filmstrip snapshots of beamforming results with the elevation angle ϕ on the X axes and the azimuth angle θ on the Y axes. It shows at what particular elevation and azimuth angles the arrivals emerge from the seabed into the water column. The first snapshot shows that the shell specular arrives at the sediment at an elevation ϕ of 147° and the azimuthal angle of 82°. This is in agreement with the expected elevation and the



FIG. 17. Focused beamforming, flush-buried target—Model data. This figure is a filmstrip of a 3-D representation of the location XY at which different beamformed target responses emerge from the seabed, while the strength of the beamformed response at that particular point is represented along the Z axis. The XY location of the flush-buried target is marked with a white vertical arrow, and the real time of the radiating shell response is marked in the upper right corner of each snapshot. Note that the arrow designates the target location, not the emerging wave location. The left snapshot shows that at 24.9414 ms the specular arrival emerges at a seabed spot whose center corresponds to the coordinate (X, Y) pair of (-2, -0.2) m. The physical significance of this result is related to the fact that this particular point on the seabed corresponds to a particular elevation and azimuth angle pair (ϕ_{spec} , θ_{spec}) at which the specular response is scattered from the buried elastic shell. The snapshot on the right shows the beamformed radiation strength due to the A0II return, which is seen to arrive at (X, Y)=(-1.6, 0) m and its own azimuth-elevation pair (ϕ_{A0II} , θ_{A0II}).

azimuth of the specular arrival in the backscattering direction from the geometric configuration of the target field.

The next snapshot on the right shows that the shell compressional return SOII arrives at almost the same azimuth, but very different elevation, namely $\phi = 155^{\circ}$. It should be noted that the higher elevation angle from the HLA to a point on the sediment surface means a shallower elevation angle from the target to that point of waveform emanation on the sediment surface. Therefore, this snapshot shows that the elastic shell compressional wave emerges from the sediment at an 8° angle difference with respect to the target specular return. In the experimental result in Fig. 16 the difference between the specular and the compressional returns was 7°.

The next snapshot shows the elevation of the S0Ix2 wave, which arrives at $\phi = 151^{\circ}$, which is also shallower than

the specular arrival. The next snapshot on the right shows that the elevation angle of the first flexural A0I arrival is 154° , which also makes it shallower than the specular arrival. In the following snapshot the elevation angle of the flexural A0II arrival is 153° , and the one next to it is its next revolution arrival.

Therefore, much like the experimental data, the model consistently predicts flexural waves emanating at shallower angles than the corresponding shell specular return. However, in the model predictions the differences between the specular and the flexural angles are higher than what is observed with the experimental data. In addition, the focused beamforming processing seems to have been more effective on the experimental data where the point of emanation of the various waves is more concentrated around a certain



FIG. 18. Elevation-azimuth focused beamformed results, flush-buried target-Model data. This figure shows at what particular elevation and azimuth angles the target-radiated arrivals emerge from the seabed into the water-column. The purple circle on the snapshots marks the (ϕ, θ) location of the target with respect to the subarray center. Specular, compressional S0, and flexural A0 waves and their multiples are identified on different time snapshots of the figure. The first snapshot shows that the shell specular arrives at the sediment at an elevation ϕ of 147° and the azimuthal angle of 82°. The next snapshot on the right shows that the shell compressional return SOII arrives at roughly the same azimuth but very different elevation, namely $\phi = 155^{\circ}$, an 8° shallower angle than the target specular return. The figure also shows that the elevation angle of the first flexural A0I arrival is 154° and A0II is 153°, which makes the flexural waves also shallower than the specular target return.

elevation/azimuth pair, while the model-focused beamforming region is broader and more ambiguous, possibly related to the modeled time series shape, suggesting that the focusing was more successful on the experimental data set. Another point is somewhat of a difference in the horizontal angle of the array between the modeling and the experimental focused beamforming geometry. While the model-focused beamforming, which is an involved procedure, was done for the original array configuration, it was later established that the array orientation has somewhat changed during this particular phase of the experiment. Nevertheless, both of the array orientations were properly taken into account, and none of the focused beamforming conclusions drawn from the model were affected by this difference.

This result now quantitatively confirms the hypothesis about the physics of the propagation of elastic waves under evanescent insonification, showing that the nature of the physical processing taking place cannot be described using the traditional wave-tracing arguments that assume all of the wave paths as to and from the target, rendering them inadequate. Furthermore, related to the detection of fully buried targets, which was so far considered a challenging problem, here-shown specifics of the structural waves' radiation from the shell, propagation through the sediment, and coupling back into the water column as propagating waves provide a clear and an efficient way of so-called "smoke-shifting" these, otherwise hardly detectable, targets.

V. CONCLUSIONS

This paper has addressed the fundamental physics associated with scattering from targets buried in a shallow water seabed, specifically addressing and confirming earlier stated hypotheses regarding the dominant scattering mechanisms and the role of structural waves in shaping the acoustic signatures of such targets. A new hybrid modeling framework for complex targets in stratified waveguides was applied to generate controlled synthetics that were passed through the same processing chain as the experimental data, allowing for identification of physical mechanisms associated with the observed signal features. Time-frequency and array processing methods are developed and applied for extracting properties of buried target signatures that can be used to classify the targets based on their reradiated returns. A theoretical timeof-arrival tool was created and implemented to obtain the expected times of arrival of specular and elastic responses of buried elastic targets and therefore identify the target waveforms. In addition, a new focused beamforming approach that was formulated and implemented was used to determine the elevation angles at which specular and elastic returns emerge from the seabed into the water column.

Through the analysis of GOATS98 experimental data, validation of target scattering models, and hypothesis validation, the frequency and the amplitude content as well as the times of arrival of target elastic response have been examined. Among the most notable results is the finding of a difference in the frequency content of clockwise and counterclockwise Lamb wave components under subcritical insonification. New identification points for bistatic classification of buried targets were presented by pointing at the amplitude and frequency differences between the specular, the clockwise, and the counter-clockwise components of Lamb waves.

Even though the desired high-pass filtering effects and the elevation angles of different wave types are both shown in the model and the experimental data, discrepancies between the model and experimental results were observed. A few possible explanations of these discrepancies, to be investigated in the future, are shear and porosity effects not included in the fluid sediment model. As a result of sediment shear effects, energy is radiated into the shear waves, which are much slower than the flexural wave field, causing the discrepancy between the experimental and model result. In theory, the virtual source approach employed by the model can be generalized to elastic media, but the procedure is intricate.

Using focused beamforming, specular and elastic arrivals were identified and their associated radiation angles were estimated, providing evidence for an earlier stated hypothesis regarding the role of structural Lamb waves in converting the evanescent insonification to a radiating scattered field. Furthermore, regarding the detection of fully buried targets, which was so far considered a challenging problem, hereshown specifics concerning the structural waves provide a clear and an efficient way of so-called "smoke-shifting" these barely detectable targets.

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